# Measuring Triple Gauge Boson Couplings at $e^+e^-$ linear Colliders

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# Outline

- Why do we measure triple gauge boson couplings
- Effective lagrangian, dimension 6 operators, and $TGC$
- Optimal observable method
- $WW\gamma$ couplings via $e^+e^- \rightarrow \nu_e\bar{\nu}_e\gamma$
- $WWZ$ couplings via $e^+e^- \rightarrow \nu_e\bar{\nu}_e Z$
- Conclusion

- Why do we measure Triple Gauge-boson Couplings
  - Test the non-Abelian nature of the electroweak sector of the SM
  - Probe for new physics effects via the TGC anomaly at ILC

- How ?
  Use an Effective Lagrangian of the SM particles

$$\mathcal{L}_{eff} = \mathcal{L}_{\rm SM} + \sum_i \frac{f_i^{(6)}\,\mathcal{O}_i^{(6)}}{\Lambda^2}\,.$$

$\Lambda$ may characterize the new physics scale, and $f_i$ are the dimensionless coeffients.

## There are 8 gauge invariant bosonic dim. 6 operators with the Higgs boson:

| $\mathcal{O}$ | | $WW$ | $ZZ$ | $Z\gamma$ | $\gamma\gamma$ | HH | $WWV$ | $HWW$ | $HZZ$ | $HZ\gamma$ | $H\gamma\gamma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{O}_{WW}$ | $\Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$ | – | – | – | – | | – | $\surd$ | $\surd$ | $\surd$ | $\surd$ |
| $\mathcal{O}_{BB}$ | $\Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi$ | | – | – | – | | | | $\surd$ | $\surd$ | $\surd$ |
| $\mathcal{O}_{BW}$ | $\Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$ | | $\surd$ | $\surd$ | $\surd$ | | $\surd$ | $\surd$ | $\surd$ | $\surd$ | $\surd$ |
| $\mathcal{O}_{W}$ | $(D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi)$ | | | | | | $\surd$ | $\surd$ | $\surd$ | $\surd$ | |
| $\mathcal{O}_{B}$ | $(D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi)$ | | | | | | $\surd$ | | $\surd$ | $\surd$ | |
| $\mathcal{O}_{\phi,1}$ | $[(D_\mu\Phi)^\dagger\Phi][\Phi^\dagger(D^\mu\Phi)]$ | | $\surd$ | | | – | | $\surd$ | $\surd$ | | |
| $\mathcal{O}_{\phi,4}$ | $(\Phi^\dagger\Phi)[(D_\mu\Phi)^\dagger(D^\mu\Phi)]$ | – | – | | | – | | $\surd$ | $\surd$ | | |
| $\mathcal{O}_{\phi,2}$ | $\frac{1}{2}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$ | | | | | – | | $\surd$ | $\surd$ | | |

$\surd$ denotes the nonstandard interactions

– denotes the no observable effects after renormalization.

# $\mathrm{WW}\gamma$ and $\mathrm{WWZ}$ couplings:

$$\begin{aligned}\mathcal{L}_{eff}^{\mathrm{WWV}} &= \mathcal{L}_{\mathrm{SM}}^{\mathrm{WWV}} + \frac{1}{\Lambda^2}\Big( f_{\mathrm{BW}}\, \Phi^\dagger \hat{\mathrm{B}}_{\mu\nu}\hat{W}^{\mu\nu}\Phi \ + f_{\mathrm{WW}}\, \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi \\ &+ f_{\mathrm{W}}\, (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi) \ + f_{\mathrm{B}}(D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi)\Big) \\ &= -ig_{\mathrm{WWV}}\left( g_1^{\mathrm{V}}(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu})V^\nu + \kappa_{\mathrm{V}}\, W^+_\mu W^-_\nu V^{\mu\nu}\right) .\end{aligned}$$

$g_{\mathrm{WW}\gamma} = e$ and $g_{\mathrm{WWZ}} = e\cot\theta_{\mathrm{W}}$.

$$g_1^\gamma = 1\,, \qquad \kappa_\gamma = 1 + (f_{\mathrm{B}} + f_{\mathrm{W}})\,\frac{m_W^2}{2\Lambda^2}\,, \qquad \kappa_{\mathrm{Z}} = 1 + \frac{m_Z^2}{2\Lambda^2}\left(\cos^2\theta_{\mathrm{W}} f_{\mathrm{W}} - \sin^2\theta_{\mathrm{W}} f_{\mathrm{B}}\right)\,, \qquad g_1^{\mathrm{Z}} = 1 + f_{\mathrm{W}}\,\frac{m_Z^2}{2\Lambda^2}$$

We examine how accurately one can measure

$$c_1 \equiv \Delta\kappa_\gamma = \kappa_\gamma - 1\,, \qquad c_2 \equiv \Delta\kappa_{\mathrm{Z}} = \kappa_{\mathrm{Z}} - 1\,, \qquad c_3 \equiv \Delta g_1^{\mathrm{Z}} = g_{\mathrm{Z}}^1 - 1$$

at ILC.

- Optimal observable method

$$\frac{d\sigma}{d\Omega} = \Sigma_{\rm SM}(\Omega) + \sum_i c_i F_i(\Omega) \;, \qquad (c_1 = \kappa_\gamma - 1, \qquad c_2 = \kappa_{\rm Z} - 1 \;, \qquad c_3 = g_1^{\rm Z} - 1)$$

Number of events in the k'th bin of the phase space point $\Omega_k$ with the bin size $\Delta\Omega$:

$$\begin{aligned} N_k^{th}(c_1, c_2, \ldots, c_n) &= L\ \Sigma_{\rm SM}(\Omega_k)\Delta\Omega + L\ \sum_i c_i F_i(\Omega_k)\Delta\Omega \\ &= N_k^{exp} \pm \sqrt{N_k^{exp}} \end{aligned}$$

Constraints on $(c_1, c_2, \ldots, c_n)$ from the data:

$$\begin{aligned} \chi^2(c_1, c_2, \ldots, c_n) &= \sum_k \left( \frac{N_k^{th} - N_k^{exp}}{\sqrt{N_k^{exp}}} \right)^2 = \sum_k \left( \frac{L\ \Sigma_{\rm SM}(\Omega_k) + L\ \sum_i c_i F_i(\Omega_k)\Delta\Omega - N_k^{exp}}{\sqrt{N_k^{exp}}} \right)^2 \\ &= \sum_k \left( \frac{L\ \sum_i c_i F_i(\Omega_k)\Delta\Omega}{\sqrt{L\ \Sigma_{\rm SM}(\Omega_k)\Delta\Omega}} \right)^2 \equiv \sum_{i,j} c_i (V^{-1})_{ij} c_j \end{aligned}$$

$$\chi^2(c_1, c_2, \ldots, c_n) \equiv \sum_{i,j} c_i (V^{-1})_{ij} c_j$$

$$(V^{-1})_{ij} = L \int \frac{F_i(\Omega) F_j(\Omega)}{\Sigma_{\rm SM}(\Omega)} d\Omega \qquad i, j = 1, 2, \ldots, n$$

$V_{ij}$ is called covariance matrix. The diagonal elements $V_{ii}$ is called variance of $c_i$. And square root of the variance gives the error of the measurement

$$\Delta c_i = \sqrt{V_{ii}}\ , \qquad \rho_{ij} = \sum_{i,j}^{n} \frac{V_{ij}}{\sqrt{V_{ii} V_{jj}}}$$

where $\rho_{ij}$ is the correlation between the error on $c_i$ and that on $c_j$.

- WW$\gamma$ couplings via $e^-e^+ \to \nu_e \bar{\nu}_e \gamma$
  differential cross section

$$\frac{d\sigma}{dP_{\mathrm{T}} d\eta} = \Sigma_{\mathrm{SM}}(P_{\mathrm{T}}, \eta) + c_1 F_1(P_{\mathrm{T}}, \eta), \qquad c_1 = \kappa_\gamma - 1$$

Inverse of the variance of $c_1$ is

$$(V^{-1})_{11} = L \sum_k \frac{F_1^2(P_{\mathrm{T}}, \eta)_k}{\Sigma_{\mathrm{SM}}(P_{\mathrm{T}}, \eta)_k} \Delta P_{\mathrm{T}} \Delta \eta \ , \quad \Delta c_1 = \sqrt{V_{11}} \ ,$$

# Background processes

$P_{\rm T}$ and $\eta$ distr. of the photon with $|\cos\theta_{e^\pm}| > 0.995$, $|\cos\theta_\gamma| < 0.995$, $\sqrt{s} = 250GeV$ for

# $P_{\mathrm{T}}$ and rapidity distributions of the photon for $e^+e^- \to \nu_e\bar{\nu}_e\gamma$

At $\sqrt{s} = 250GeV$ we obtain

$$\int \Sigma_{\rm SM}(P_{\rm T},\eta) dP_{\rm T} d\eta \approx 5.5 \times 10^4 fb\ , \quad V^{-1} = L \int \frac{F_1^2(P_{\rm T},\eta)}{\Sigma_{SM}(P_{\rm T},\eta)} dP_{\rm T} d\eta \approx L \times 50 fb$$

$$\Delta c_1 = \sqrt{V} \approx 0.14/\sqrt{L(fb^{-1})}$$

We find for various $\sqrt{s}$ and $L$:

| $\Delta(\kappa\gamma - 1)$ | $\sqrt{s}(GeV)$ | $L(fb^{-1})$ | $colliders$ |
|---|---|---|---|
| $\sim 1$ | 183 | $0.08$ | LEP2 |
| $0.014$ | 250 | $100$ | ILC |
| $0.0090$ | 350 | $100$ | .. |
| $0.0034$ | 500 | $300$ | ILC |
| $0.0015$ | 1000 | $500$ | ILC2 |

Discussion:

We observe that as the cross section increases for higher and higher $\sqrt{s}$, the error in measuring these coupling parameters becomes smaller and smaller.

- WWZ couplings via $e^+e^- \to \nu_e\bar{\nu}_e Z$

There are two terms in the WWZ couplings : $\kappa_{\mathrm{Z}} - 1 = c_2$ , $g_1^{\mathrm{Z}} - 1 = c_3$

$$(V^{-1})_{ij} = L \int \frac{F_i(\Omega)F_j(\Omega)}{\Sigma_{\mathrm{SM}}(\Omega)} d\Omega \qquad i,j = 2,3$$

$$\Delta c_i = \sqrt{V_{ii}} , \quad i = 2,3 , \qquad \rho_{23} = \frac{V_{23}}{\sqrt{V_{22}V_{33}}}$$

## $P_T$ and rapidity distributions of Z boson for $e^+e^- \rightarrow \nu_e\bar{\nu}_e Z$

We find for various $\sqrt{s}$ and $L$:

| $\Delta(\kappa_Z - 1)$ | $\Delta(g_1^Z - 1)$ | $\rho(\kappa_Z - 1, g_1^Z - 1)$ | $\sqrt{s}(GeV)$ | $L(fb^{-1})$ |
|---|---|---|---|---|
| 0.035 | 0.029 | 0.31 | 250 | 100 |
| 0.014 | 0.0099 | 0.33 | 350 | 100 |
| 0.0037 | 0.0024 | 0.32 | 500 | 300 |
| 0.00085 | 0.00044 | 0.31 | 1000 | 500 |

Discussion:

We observe that as the cross section increases for higher and higher $\sqrt{s}$, the error in measuring these couplings becomes smaller and smaller.

- **Conclusion**

We obtain the parameters of the TGC's for various $\sqrt{s}$ and $L$ by studying the processes $e^+e^- \to \nu_e\bar{\nu}_e\gamma$, and $e^+e^- \to \nu_e\bar{\nu}_e Z$

| $\Delta(\kappa_\gamma - 1)$ | $\Delta(\kappa_Z - 1)$ | $\Delta(g_1^Z - 1)$ | $\rho(\kappa_Z - 1, g_1^Z - 1)$ | $\sqrt{s}(GeV)$ | $L(fb^{-1})$ |
|---|---|---|---|---|---|
| 0.014 | 0.035 | 0.029 | 0.31 | 250 | 100 |
| 0.009 | 0.014 | 0.0099 | 0.33 | 350 | 100 |
| 0.0034 | 0.0037 | 0.0024 | 0.32 | 500 | 300 |
| 0.0015 | 0.00085 | 0.00044 | 0.31 | 1000 | 500 |

- These TGC parameters are sensitive to new physics that contribute to the dim-6 operators, $\mathcal{O}_W$ and $\mathcal{O}_B$.
- $\Delta(\kappa_\gamma - 1) < \kappa_Z - 1$ for $\sqrt{s} \leq 500GeV$
- $\Delta(\kappa_\gamma - 1) > \kappa_Z - 1$ for $\sqrt{s} \leq 1TeV$ because of $\sigma(e^+e^- \to \nu_e\bar{\nu}_e Z) > \sigma(e^+e^- \to \nu_e\bar{\nu}_e\gamma)$ at higher energies.

Electromagnetic gauge invariance requires that $g_1^\gamma = 1$, while the $\kappa_\gamma$, $\kappa_{\rm Z}$ and $g_1^{\rm Z}$ are related to the coefficients of the dimension six operators as

$$\kappa_\gamma - 1 = (f_{\rm B} + f_{\rm W})\,\frac{m_W^2}{2\Lambda^2}\ , \quad \kappa_{\rm Z} - 1 = \frac{m_Z^2}{2\Lambda^2}\left(\cos^2\theta_{\rm W} f_{\rm W} - \sin^2\theta_{\rm W} f_{\rm B}\right)\ , \quad g_1^{\rm Z} - 1 = f_{\rm W}\,\frac{m_{\rm Z}^2}{2\Lambda^2}$$

We find the constraints on $f_{\rm W}$ and $f_{\rm B}$

| $\Delta f_W$ | $\Delta f_B)$ | $\rho(\kappa_{\rm Z} - 1, g_1^{\rm Z} - 1)$ | $\sqrt{s}(GeV)$ | $L(fb^{-1})$ |
|---|---|---|---|---|
| 3.1 | 13.5 | −0.08 | 250 | 100 |
| 2.0 | 5.5 | −0.18 | 350 | 100 |
| 0.56 | 1.45 | −0.20 | 500 | 300 |
| 0.11 | 0.34 | −0.22 | 1000 | 500 |

# Thank You!