Search for extra dimensions at a linear collider

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April 27, 2008

中国高能物理学会第十届全国粒子物理学术会议 南京

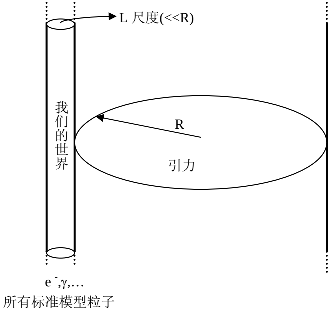
Outline

1. Extra dimension models
2. What we did
   - $e^+e^- \to HZZ/HHZ$
   - $\gamma\gamma \to e^+e^-G_n$
3. Summary and outlook

Large extra dimension(LED) model

In the LED model, the space-time dimension is $D = n + 4$, and the n dimensions are compacted to a torus with radius R.

Arkani-Hamed, Dimopoulos, and Dvali proposed that, SM particles live in 4 dimensional space, gravity can propagate in a higher-dimensional space.



The gravity potential becomes

$$V(r) \propto \frac{1}{M_S^{2+n}} \frac{m_1 m_2}{r^{1+n}} \quad , when\ r \ll R$$

$$V(r) \propto \frac{1}{M_S^{2+n} V} \frac{m_1 m_2}{r} \propto \frac{1}{M_P^2} \frac{m_1 m_2}{r}, when\ r \gg R$$

that is, the Plank scale be expressed as:

$$M_P^2 \sim R^n M_S^{2+n}$$

suppose that R is large enough to make $M_S \sim M_W$, then the hierarchy problem is naturally solved.

If $M_S = 1 TeV$, then

- n=1,$R \sim 10^{13} cm$, violate Newton's law, ruled out;
- n=2, $R \sim 1 mm$, latest result: $R \leq 44 \mu m$, ruled out;
- $n \geq 3$, $R \sim 1 nm$, testable at future colliders

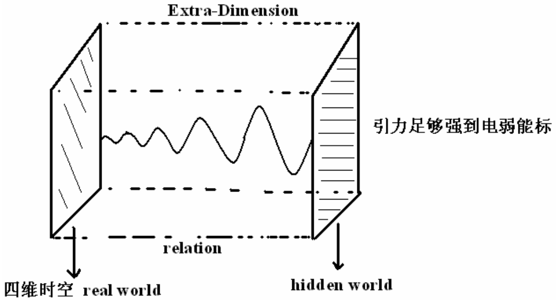
Expand the D-dimension field $h_{MN}(x^\mu, y^i)$ in its Fourier series

$$h_{MN}(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_{m \in Z^n} h_{MN}^{(m)}(x) e^{i \frac{2\pi m \cdot y}{R}}$$

This can be expressed as:

a massless graviton in D dimension → Fourier expanding → a lot of massive KK modes of graviton in 4 dimension

Randall-Sundrum(RS) model



D=4+1, $ds^2 = \exp(-2kR|\varphi|)\eta_{\mu\nu}dx^\mu dx^\nu - R^2 d\varphi^2$

$0 \leq \varphi < 2\pi$: the coordinate along the extra dimension of radius R，

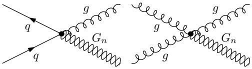
k: the curvature of the AdS space。

Two 4-dimensional branes are put at $\varphi = 0$ and $\varphi = \pi$.

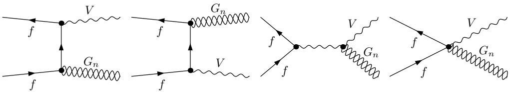
The first KK-state mass might be several hundred GeV.

How to test

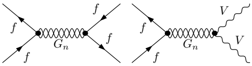
- Direct graviton production
  - monojet + missing E_T



  - Vector boson + missing E_T， used at e^+e^- collider



- Virtual graviton exchange
  - fermions/bosons pair， used at hadron and linear colliders

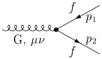


What we did

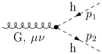
- $e^+e^- \to HZZ/HHZ$, virtual graviton exchange. Cross section calculation, limits for M_S, in e^+e^- unpolarization and polarization cases. These processes are important in testing Higgs self-coupling and Higgs coupling with gauge boson.
- $\gamma\gamma \to e^+e^-G_n$, real graviton emission. Cross section calculation, signal analysis, give limits to M_S. Also in $\gamma\gamma$ unpolarization and polarization cases. $\gamma\gamma \to l^+l^-$ is the best process for the measurement of the luminosity, so it is convenient to select events with missing energy from these beam calibration processes.

The vertices we used

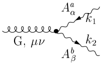
de Donder gauge



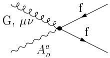
$$C_{ffG} = -i\frac{\kappa}{8}\left[\gamma_\mu(p_1+p_2)_\nu + \gamma_\nu(p_1+p_2)_\mu - 2\eta_{\mu\nu}(\not p_1 + \not p_2 - 2m_f)\right]$$



$$C_{hhG} = i\kappa(B_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta - \frac{1}{2}\eta_{\mu\nu}m_h^2)$$



$$C_{AAG} = i\kappa[B_{\mu\nu\alpha\beta}m_A^2 + (C_{\mu\nu\alpha\beta\rho\sigma} - C_{\mu\nu\alpha\sigma\beta\rho})k_1^\rho k_2^\sigma]\ \delta^{ab}$$



$$C_{ffAG} = ig\frac{\kappa}{4}(\gamma_\mu\eta_{\nu\rho} + \gamma_\nu\eta_{\mu\rho} - 2\gamma_\rho\eta_{\mu\nu})T$$

where ,

$$B^{\mu\nu\alpha\beta} = \frac{1}{2}(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}),$$

$$C^{\rho\sigma\mu\nu\alpha\beta} = \frac{1}{2}[g^{\rho\sigma}g^{\mu\nu}g^{\alpha\beta} - (g^{\rho\mu}g^{\sigma\nu}g^{\alpha\beta} + g^{\rho\nu}g^{\sigma\mu}g^{\alpha\beta} + g^{\rho\alpha}g^{\sigma\beta}g^{\mu\nu} + g^{\rho\beta}g^{\sigma\alpha}g^{\mu\nu})],$$

$$\kappa = \sqrt{16\pi G_N}$$

KK modes summation

Summation of KK modes

$$M_m = \frac{2\pi|m|}{R} \Longrightarrow \Delta M = \frac{2\pi}{R} \sim M_S \left(\frac{M_S}{\bar{M}_P}\right)^{2/n} \sim \left(\frac{M_S}{\mathrm{TeV}}\right)^{\frac{n+2}{2}} 10^{\frac{12n-31}{n}} \mathrm{eV}$$

$$\Delta M \sim \begin{cases} 20\ KeV, & 当n=4时; \\ 7\ MeV, & 当n=6时; \\ 0.1\ GeV, & 当n=8时; \\ & \ldots \end{cases}$$

Summation $\Longrightarrow$ Integral

- Summation of KK modes for real graviton
  the number of KK modes between $|m|$ and $|m| + dm$:

$$\begin{aligned} dN &= S_{n-1}|m|^{n-1} dm \quad \left(S_{n-1} = \frac{2\pi^{n/2}}{\Gamma(n/2)}\right) \\ &= \frac{R^n M_m^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)} dM_m^2 \equiv \rho(M_m)\, dM_m^2. \end{aligned}$$

differential cross section：

$$\frac{d^2\sigma}{dt\ dM_m^2} = \rho(M_m)\ \frac{d\sigma_{M_m}}{dt},$$

- Summation of KK modes for virtual graviton
  Graviton propagator :$\frac{iP_{\mu\nu\alpha\beta}}{s-M_m^2+i\epsilon}$,
  where $P^{\mu\nu\alpha\beta} = \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \frac{2}{n-2}\eta^{\mu\nu}\eta^{\alpha\beta}$

$$D(s) = \sum_m \frac{i}{s - M_m^2 + i\varepsilon} = \int_0^\infty dM_m^2\,\rho(M_m)\frac{i}{s - M_m^2 + i\varepsilon},$$

Using

$$\frac{1}{s - m^2 + i\varepsilon} = P\left(\frac{1}{s - m^2}\right) - i\pi\delta(s - m^2)$$

We obtain

$$D(s) = \frac{s^{n/2-1}}{\Gamma(n/2)}\frac{R^n}{(4\pi)^{n/2}}\left[\pi + 2iI(M_S/\sqrt{s})\right],$$

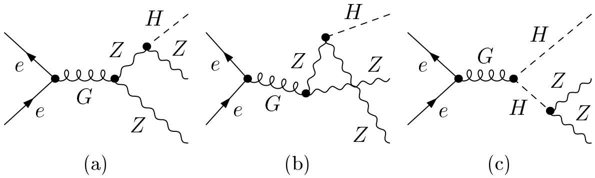
where

$$I(M_S/\sqrt{s}) \;=\; P\int_0^{M_S/\sqrt{s}} dy\,\frac{y^{n-1}}{1-y^2}\,.$$

This principal integration can be carried out as:

$$\begin{aligned} I(M_S/\sqrt{s}) &= -\sum_{k=1}^{n/2-1}\frac{1}{2k}\left(\frac{M_S}{\sqrt{s}}\right)^{2k} - \frac{1}{2}\log\left(\frac{M_S^2}{s}-1\right) \qquad n=\text{even},\\ &= -\sum_{k=1}^{(n-1)/2}\frac{1}{2k-1}\left(\frac{M_S}{\sqrt{s}}\right)^{2k-1} + \frac{1}{2}\log\left(\frac{M_S+\sqrt{s}}{M_S-\sqrt{s}}\right) \qquad n=\text{odd}.\end{aligned}$$

$e^+e^- \to HZZ$: Feynman diagrams and amplitudes



$$\mathcal{M}^G_{(a)} = \frac{g m_W D(s) P_{\rho\sigma\alpha\beta}}{2 S_W C_W (m_H^2 + 2 p_1 \cdot p_2)} \bar{v}(k_1,\lambda_1)[V_{eeG}]_{\rho\sigma} u(k_2,\lambda_2)[V_{ZZG}]_{\alpha\beta\mu\nu}\, \epsilon_\nu(p_2)\epsilon_\mu(p_3)$$

$$\mathcal{M}^G_{(b)} = \mathcal{M}^G_{(a)}\, [(p_2,\nu) \leftrightarrow (p_3,\mu)]$$

$$\mathcal{M}^G_{(c)} = \frac{-g m_W D(s) P_{\rho\sigma\alpha\beta}}{2 S_W C_W^2 ((p_2+p_3)^2 - m_H^2)} \bar{v}(k_1,\lambda_1)[V_{eeG}]_{\rho\sigma} u(k_2,\lambda_2)[V_{HHG}]_{\alpha\beta}\, g_{\mu\nu}\epsilon_\nu(p_2)\epsilon_\mu(p_3)$$

Total amplitudes： $\mathcal{M} = \mathcal{M}^G + \mathcal{M}^{SM}$

cross section

cross section with unpolarized initial beams：

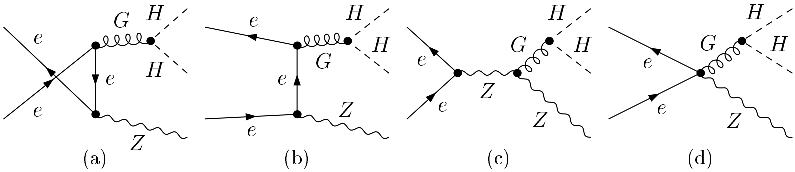
$$\sigma_{LED}^{unpol.} = \frac{1}{2|\vec{k}_1|\sqrt{s}} \int d\Phi_3 \frac{1}{4} \sum_{\lambda_1,\lambda_2} |\mathcal{M}(\lambda_1,\lambda_2)|^2,$$

after polarized：

$$\sum_{\lambda_1,\lambda_2} |\mathcal{M}(\lambda_1,\lambda_2)|^2 \longrightarrow \frac{(1+\mathcal{P}_e)}{2}\frac{(1+\mathcal{P}_p)}{2}|\mathcal{M}(+,+)|^2 + \frac{(1+\mathcal{P}_e)}{2}\frac{(1-\mathcal{P}_p)}{2}|\mathcal{M}(+,-)|^2 + \frac{(1-\mathcal{P}_e)}{2}\frac{(1+\mathcal{P}_p)}{2}|\mathcal{M}(-,+)|^2 + \frac{(1-\mathcal{P}_e)}{2}\frac{(1-\mathcal{P}_p)}{2}|\mathcal{M}(-,-)|^2$$

$\mathcal{P}_e = \frac{N_e^+ - N_e^-}{N_e^+ + N_e^-}$, $\mathcal{P}_p = \frac{N_p^+ - N_p^-}{N_p^+ + N_p^-}$, polarization efficiency

$e^+e^- \to HHZ$: Feynman diagrams and amplitudes



$$\mathcal{M}^G_{(a)} = \frac{-gD(s_1)P_{\rho\sigma\alpha\beta}}{8C_W(k_1-p_3)^2}\bar{v}(k_1,\lambda_1)\gamma_\mu(1-4S_W^2-\gamma_5)(\not{k}_1-\not{p}_3)[V_{eeG}]_{\rho\sigma}k(k_2,\lambda_2)[V_{HHG}]_{\alpha\beta}\epsilon_\mu(p_3)$$

$$\mathcal{M}^G_{(b)} = \frac{-gD(s_1)P_{\rho\sigma\alpha\beta}}{8C_W(k_2-p_3)^2}\bar{v}(k_1,\lambda_1)[V_{eeG}]_{\rho\sigma}(\not{k}_2-\not{p}_3)\gamma_\mu(1-4S_W^2-\gamma_5)u(k_2,\lambda_2)[V_{HHG}]_{\alpha\beta}\epsilon_\mu(p_3)$$

$$\mathcal{M}^G_{(b)} = \frac{gD(s_1)P_{\rho\sigma\alpha\beta}}{8C_W(s-m_Z^2)}\bar{v}(k_1,\lambda_1)\gamma_\nu(1-4S_W^2-\gamma_5)u(k_2,\lambda_2)[V_{ZZG}]_{\rho\sigma\mu\nu}[V_{HHG}]_{\alpha\beta}\epsilon_\mu(p_3)$$

$$\mathcal{M}^G_{(b)} = \frac{D(s_1)P_{\rho\sigma\alpha\beta}}{2}\bar{v}(k_1,\lambda_1)[V_{eeZG}]_{\rho\sigma\mu}u(k_2,\lambda_2)[V_{HHG}]_{\alpha\beta}\epsilon_\mu(p_3)$$

Total amplitudes： $\mathcal{M} = \mathcal{M}^G + \mathcal{M}^{SM}$

Input parameters

$$\mathcal{P}_e = 0.8, \mathcal{P}_p = 0.6$$

$$\mathcal{L}_{e^+e^-} = 500 fb^{-1}$$

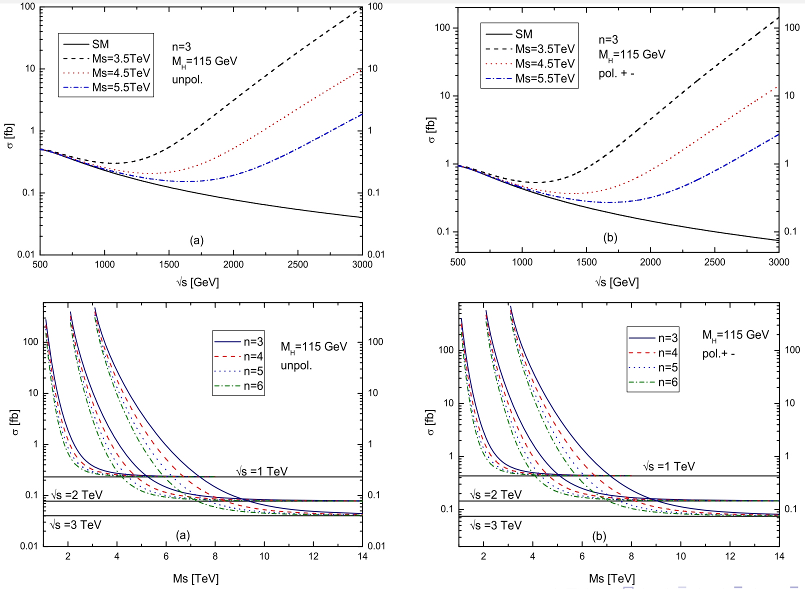
$$\alpha = 1/127.918$$

$$S_W^2 = 0.2312$$

$$M_z = 91.1876 GeV$$

$$|\cos\theta_{final}| < 0.966$$

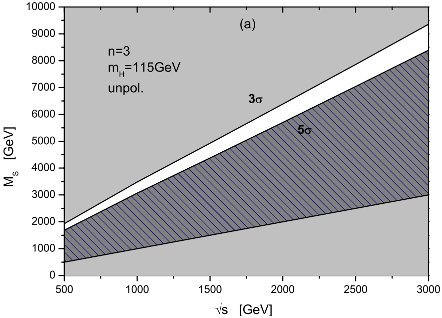
$e^+e^- \to HZZ$: numerical results and analysis



$e^+e^- \to HZZ$: discovery and exclusion limits on M_S

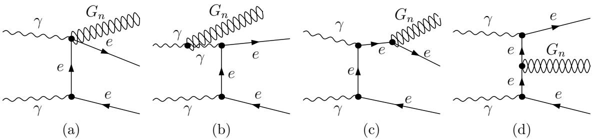
$$\Delta\sigma = \sigma_{LED} - \sigma_{SM} \geq \frac{5\sqrt{\sigma_{LED}\mathcal{L}}}{\mathcal{L}}, \quad ED \text{ discovery limit}$$

$$\Delta\sigma = \sigma_{LED} - \sigma_{SM} \leq \frac{3\sqrt{\sigma_{LED}\mathcal{L}}}{\mathcal{L}}, \quad ED \text{ exclusion limit}$$



m_h = 115[GeV]	M_S [GeV]			
$\sqrt{s}$	*unpol.*		*pol.* + -	
[TeV]	3σ	5σ	3σ	5σ
0.5	1930,	1683	2029,	1771
1.0	3483,	3068	3654,	3213
1.5	4927,	4378	5136,	4559
2.0	6383,	5700	6623,	5917
2.5	7861,	7039	8129,	7296
3.0	9355,	8392	9662,	8691
3.5	10867,	9751	11214,	10098

$\gamma\gamma \to e^+e^-G_n$: Feynman diagrams, amplitudes, cross section



Spin-2 graviton
no spin-0 because $\sigma_{spin-0} \propto M_e^2 \sim 0$

$$\mathcal{M}_a = \frac{ie^2\kappa}{4}\frac{1}{(k_5-p_2)^2}\epsilon_{\nu_1}(p_1)\epsilon_{\nu_2}(p_2)\epsilon_{\mu_1\mu_2}(k_3)\bar{u}(k_4)(\gamma_{\mu_2}\eta_{\mu_1\nu_1}+\gamma_{\mu_1}\eta_{\mu_2\nu_1}-2\gamma_{\nu_1}\eta_{\mu_1\mu_2})$$
$$(\not{p}_2-\not{k}_5)\gamma_{\nu_2}v(k_5),$$

......

squared amplitudes：

$$\overline{\sum_{spins}} |\mathcal{M}|^2 = \frac{1}{4} \sum_{spins} \left(\sum_{i=1}^{14} \mathcal{M}_i \right)^\dagger \left(\sum_{i=1}^{14} \mathcal{M}_i \right)$$

Spin-sum of the polarization tensors:

$$\sum_{\lambda_s=1}^{5} \epsilon_{\mu\nu}(k,\lambda_s)\epsilon^*_{\alpha\beta}(k,\lambda_s) = P_{\mu\nu\alpha\beta}(k).$$

where

$$\begin{aligned} P_{\mu\nu\alpha\beta} \quad = \quad & \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right) \\ & -\frac{1}{2m^2}\left(\eta_{\mu\alpha}k_\nu k_\beta + \eta_{\nu\beta}k_\mu k_\alpha + \eta_{\mu\beta}k_\nu k_\alpha + \eta_{\nu\alpha}k_\mu k_\beta\right) \\ & +\frac{1}{6}\left(\eta_{\mu\nu} + \frac{2}{m^2}k_\mu k_\nu\right)\left(\eta_{\alpha\beta} + \frac{2}{m^2}k_\alpha k_\beta\right). \end{aligned}$$

KK modes summation to integral:

$$\sigma = \sum_n \sigma_m \to \int_0^{\sqrt{s}} \rho(m)\, \sigma_m \, dm$$

input parameters and notations

- $\alpha = 1/128$
- $2^\circ < \theta_{e\gamma} < 178^\circ$
- polarization

  Incoming $\gamma\gamma$ polarization modes: $+-$,$-+$,$++$,$--$, unpolarized

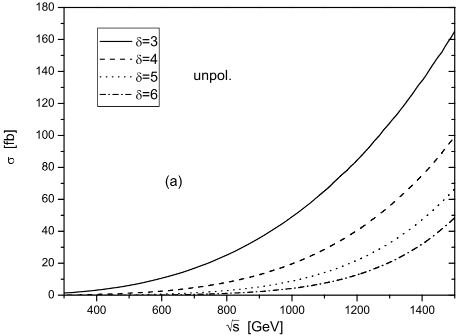
  + - represents helicities of photons being $\lambda_1 = 1$, $\lambda_2 = -1$,

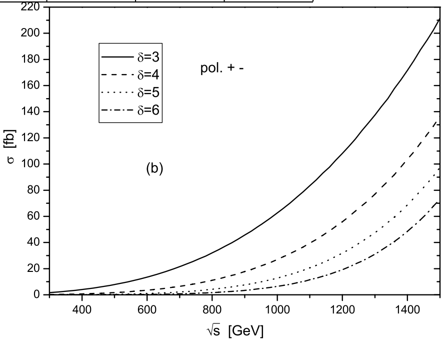
  Because $|\mathcal{M}(++)|^2 = |\mathcal{M}(--)|^2$, $|\mathcal{M}(+-)|^2 = |\mathcal{M}(-+)|^2$,we only consider $++$ and $-+$ cases

  polarization efficiency $P_\gamma (P_\gamma \equiv \frac{N_+ - N_-}{N_+ + N_-})$ set to be 0.9

numerical results of cross sections

$\sqrt{s}$ [GeV]		$\delta = 3$	$\delta = 4$	$\delta = 5$	$\delta = 6$
500	unpolarized	46.46	13.92	4.692	1.700
	$+-$	60.01	19.35	6.853	2.576
	$++$	32.91	8.493	2.532	0.821
1000	unpolarized	371.7	222.7	150.1	108.8
	$+-$	480.8	309.6	219.3	164.9
	$++$	262.6	135.8	80.93	52.75





signal analysis

Signature for this process: $\gamma\gamma \to e^+e^- + \textit{missing energy}$.
Main background at the lowest order:

$$\gamma\gamma \to e^+e^- \longrightarrow \text{totally removed by a cut on } \theta_{e^+e^-}$$
$$\gamma\gamma \to e^+e^-Z \to e^+e^-(\nu\bar{\nu})$$
$$\gamma\gamma \to W^+W^- \to (e^+\nu_e)(e^-\bar{\nu}_e)$$
$$\gamma\gamma \to \tau^+\tau^- \to (e^+\nu_e\bar{\nu}_\tau)(e^-\bar{\nu}_e\nu_\tau)$$

Contributions from these backgrounds:

$$\sigma_{e^+e^-Z} = \sigma(\gamma\gamma \to e^+e^-Z) \times Br(Z \to \nu\bar{\nu}) = \begin{cases} 10.65\ fb & (for\ \sqrt{s} = 500\ GeV); \\ 4.17\ fb & (for\ \sqrt{s} = 1000\ GeV). \end{cases}$$

$$\sigma_{WW} = \begin{cases} 1011\ fb & (for\ \sqrt{s} = 500\ GeV) \\ 1019\ fb & (for\ \sqrt{s} = 1000\ GeV). \end{cases}$$

$$\sigma_{\tau\tau} = \begin{cases} 243.1\ fb & (for\ \sqrt{s} = 500\ GeV) \\ 62.36\ fb & (for\ \sqrt{s} = 1000\ GeV). \end{cases}$$

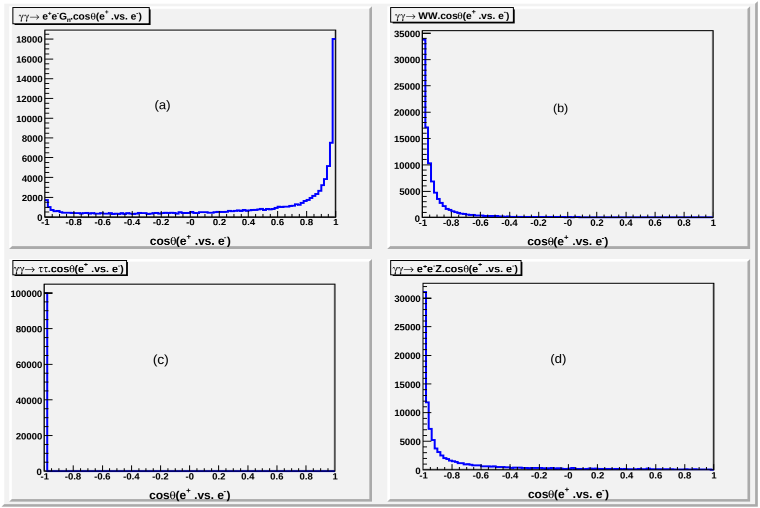


Figure: Distributions of the open angel between the electron and positron for the signal process when extra dimensions $\delta = 3$ and the background processes. The $\gamma\gamma$ c.m.s. energy is 1 TeV and $\mathrm{M_S}$ is set to be 1 TeV.

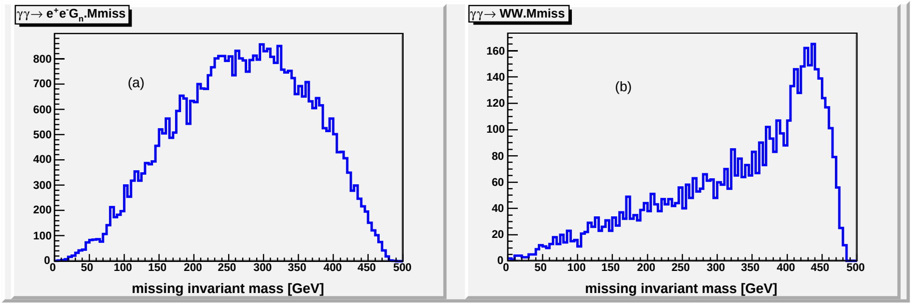


Figure: Distributions of the missing invariant mass of the signal process and the WW-background process after applying $CUT1$ when extra dimensions $\delta = 3$. The $\gamma\gamma$ c.m.s. energy is 500 GeV and $\mathrm{M_S}$ is set to be 1 TeV.

Events selection strategies

1. For $\sqrt{s} = 500$ and $\sqrt{s} = 1000GeV$,
   - Take into the detector acceptance:
     - $5^\circ < \theta_{e\gamma} < 175^\circ$;
     - $p_T^e > 5$ GeV;
     - $E_e > 1GeV$.
     - $\theta_{ee} > 5^\circ$.
   - To eliminate the WW, $\tau\tau$, $\gamma\gamma \to e^+e^-$ and e^+e^-Z backgrounds, $5^\circ < \theta_{ee} < \theta_{ee}^{cut} = 90^\circ$。

   denote these cuts as $CUT1$ set:

   $$5^\circ < \theta_{e\gamma} < 175^\circ,\ p_T^e > 5\ GeV,\ E_e > 1\ GeV, \text{ and } 5^\circ < \theta_{ee} < \theta_{ee}^{cut} = 90^\circ$$

2. when $\sqrt{s} = 500$,
   one more cut needed, denoted as $CUT2$:

   $$M_{miss} < 400\ GeV$$

events selection results

Table: Event selection on background and signal($\delta = 3$) with unpolarization case.

	$\sqrt{s} = 500GeV$				$\sqrt{s} = 1000GeV$			
	$e^+e^-G_n$	WW	$\tau\tau$	e^+e^-Z	$e^+e^-G_n$	WW	$\tau\tau$	e^+e^-Z
N before cut	4646	101100	24310	1065	37170	101900	6236	417
N after $CUT1$	1805	5402	0	86	17790	649	0	31
N after $CUT2$	1616	3427	0	86	/	/	/	/
efficiency ϵ	34.8%	3.39%	0%	8.08%	47.9%	0.64%	0%	7.43%
SB	27.26				682.2			

Table: Event selection on background and signal($\delta = 3$),with $+-$ polarized photon beams($P_\gamma = 0.9$).

	$\sqrt{s} = 500GeV$				$\sqrt{s} = 1000GeV$			
	$e^+e^-G_n$	WW	$\tau\tau$	e^+e^-Z	$e^+e^-G_n$	WW	$\tau\tau$	e^+e^-Z
N before cut	5926	96159	38271	1340	47400	99065	10299	480
N after $CUT1$	2104	5152	0	59	21524	631	0	19
N after $CUT2$	1782	3268	0	59	/	/	/	/
efficiency ϵ	30.1%	3.40%	0%	4.40%	45.4%	0.64%	0%	3.96%
SB	30.89				844.2			

limits on M_S

significance of signal over background(SB):

$$SB = \frac{N_{signal}}{\sqrt{N_{background}}} = \frac{\sigma_S^{CUT} \cdot \mathcal{L}_{\gamma\gamma}}{\sqrt{\sigma_B^{CUT} \cdot \mathcal{L}_{\gamma\gamma}}} = \frac{\sigma_S^{CUT}}{\sqrt{\sigma_B^{CUT}}} \cdot \sqrt{\mathcal{L}_{\gamma\gamma}}$$

$\because \sigma \propto 1/M_S^{\delta+2}$, $\therefore SB \propto 1/M_S^{\delta+2}$

suppose that the signature can be detected only when $SB \geq 5$, then with $\delta = 3$,

$\sqrt{s} = 1TeV$	unpol.	$M_s \leq 2.67$
	pol.	$M_s \leq 2.79$
$\sqrt{s} = 500GeV$	unpol.	$M_s \leq 1.40$
	pol.	$M_s \leq 1.44$

Summary and outlook

- We considered some processes at future linear colliders, calculated the cross sections and exclusion limits on M_S. For the graviton emission process, we made some signal analysis and present strategies to distinguish graviton from background.
- More research can be done in ADD model and also RS model, including QCD corrections, at pp colliders, et.al.

Thank you