Dispersive analysis of $\gamma\gamma \longrightarrow \pi\pi$ process

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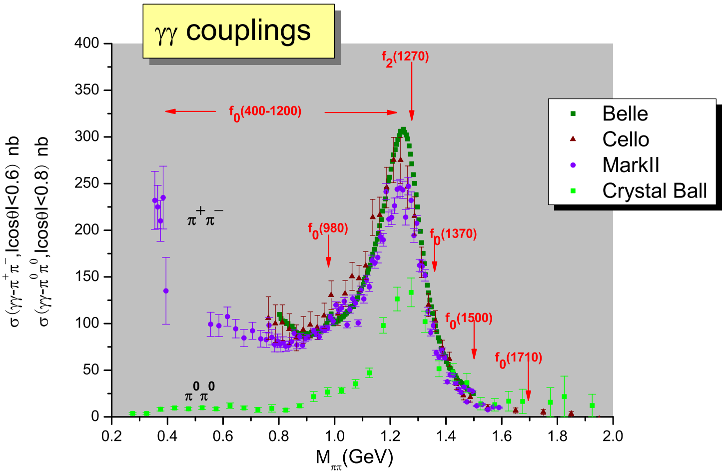


Figure: Limited angular coverage. No polarization

- In an ideal world, with complete information on all the possible angular correlations between the initial and final state directions and spins, we could decompose the cress-sections into components with definite sets of quantum numbers.
- Unfortunately, in the real world, experiments have only a limited angular coverage and the polarization of the initial state is not measured. This lack of information plays a crucial role in any analysis and affects the determination of the resonance couplings.

- M.R.Pennington *et al.*

For each partial amplitude with definite spin J, helicity λ and isospin I unitarity requires:

$$ImF^{I}_{J\lambda}(\gamma\gamma \to \pi\pi) = \sum_n \rho_n F^{I}_{J\lambda}(\gamma\gamma \to \ h)^* T^{I}_{J}(h \to \pi\pi)$$

However, each hadronic amplitude $T^{I}_{J}(h \to \pi\pi)$ satisfies the non-linear unitarity relation:

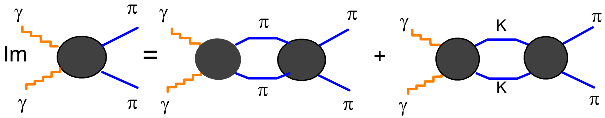
$$ImT^{I}_{J}(h \to \pi\pi) = \sum_{h'} \rho_{h'} T^{I}_{J}(h \to \ h')^* T^{I}_{J}(h' \to \pi\pi)$$

This equation means that

$$F^{I}_{J\lambda}(\gamma\gamma \to \pi\pi) = \sum_h \overline{\alpha}^{I,J\lambda}_{h} T^{I}_{J}(h \to \pi\pi)$$

Below 1.4GeV, where $\pi\pi$ and $K\overline{K}$ channels are essentially all that are relevant, then:

$$F^{I}_{J\lambda}(\gamma\gamma \to \pi\pi) = \overline{\alpha}^{I,J\lambda}_{\pi} T^{I}_{J}(\pi\pi \to \pi\pi) + \overline{\alpha}^{I,J\lambda}_{K} T^{I}_{J}(K\overline{K} \to \pi\pi)$$



•J.A.Oller *et al.*

L_I is the complete left hand cut contribution. Then $F_I - L_I$ only has right hand cut. Consider the Omnés function $\omega_I(s)$

$$\omega_I(s) = exp[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\phi_I(s^{'})}{s^{'}(s^{'}-s)}ds^{'}]$$

Then perform a twice subtracted dispersion relation for $\frac{(F_I(s)-L_I(s))}{\omega_I(s)}$

$$F_I(s) = L_I(s)+a_I\omega_I(s)+c_I s\omega_I(s)+\frac{s^2}{\pi}\omega_I(s)\int_{4m_\pi^2}^{\infty}\frac{L_I(s^{'})\sin\phi_I(s^{'})}{s^{'2}(s^{'}-s)|\omega_I(s^{'})|}ds^{'}$$

Low's theorem requires $F_I \rightarrow\ B_I(s)$, for $s \rightarrow 0$ if we write $R_i \rightarrow 0, for\ s \rightarrow\ 0\quad L_I = B_I + R_I$, it implies that $F_I - L_I \rightarrow\ 0$, for, $s \rightarrow 0$and hence $a_I = 0$

•Charged Pion-Pair Production and Pion Polarizabilities to two Loops (U.Burgi) arXiv: hep-ph/9602429vl. 29 Feb.1996

•Revisiting$\gamma\gamma \rightarrow \pi^+\pi^-$ at low energies (J.Gasser,M.A.Ivanov,,M.E.Sainio) Nuclear Physics B 745 (2006) 84-108

•Low-energy photon-hotoon collisions to two loops revisited (J.Gasser,M.A.Ivanov,M.E.Sainio) arXiv:hep-ph/0506265vl 25 Jun 2005

•Determination of π^0 meson quarupole polarizabilities from the process $\gamma\gamma \rightarrow \pi^0\pi^0$ (L.V.Fil'kov. and V.L.Kashevarov) arXiv: nucl-th/0505058v3 5Jul 2005

•Determination of $\pi^\pm$ meson polarizabilities from the $\gamma\gamma \rightarrow \pi^+\pi^-$ process (L.V.Fil'kov. and V.L.Kashevarov.) arXiv: nucl-th/0512047. v2 2Apr 2006

•Lightest scalar and tensor resonances in $\gamma\gamma \rightarrow \pi\pi$ after the Bele experiment (N.N.Achasov, G N.Shestakov) arXiv: 0712.0885vl [hep-ph]. 6 Dec 2007

•Lightest Scalar in $SU_L(2) \times SU_R(2)$ Linear σ Model
(N.N.Achasov and G.N.shestakov) PRL.99, 072001(2007)
•Shape of the $f_0(980)$ in $\gamma\gamma \to \pi^+\pi^-$
(N.N.Achasov and G.N.Shestakov) PHYSICAL REVIEW D 72, 013006(2005)
•TWO-PION PRODUCTION IN PHOTON-PHOTON COLLISIONS
(J. BIJNENS and F. CORNET) Nuclear Physics B296(1988)557-568
•Photon-Photon Scattering, Pion Polarizability and chiral Symmetry
(J. F.Donoghue and B. R.Holstein) arXiv: hep-ph/9302203vl. 2 Feb 1993
•SOME COMMENTS ON PION-PAIR PRODUCTION IN LOW ENERGY PHOTON-PHOTON COLLISIONS
(G. MENNESSIER and T. N.TRUONG) PHYSICS LETTERS B. Volume177 number2. 11 September 1986
•Meson Pair Production in $\gamma\gamma$ Scattering and Implications for Scalar Mesons
(G. Mennessier) Z. Phys. C-Particles and Fields 16 241-255(1983)

- Study of the reactions $\gamma\gamma \to \pi\pi$ is able to benefit from two key simplifications: the existence of a relevant low energy theorem and predictable final state interactions. The strict requirement is for the $\gamma\gamma \to \pi\pi$ amplitudes to their one-pion-exchange(OPE) Born approximation at the crossed-channel threshold. And these Born amplitudes, modified only by known final state interactions, control the $\gamma\gamma \to \pi\pi$ amplitude for a considerable energy range above threshold. Indeed, for the I=0,2 S and D waves this is up to some 600-MeV, while for all the higher waves (including incidentally the I=2 D wave) known final state interactions are sufficiently weak phase shifts are less than a few degrees-that their Born approximation is excellent throughout the energy region we study, up to 1.4GeV.
- It is these key properties of the low energy Born approximations and unitarity for the specific process $\gamma\gamma \to \pi\pi$ that makes an amplitude analysis of data with such limited angular coverage possible.

The unpolarized cross-section is thus written

$$\frac{d\sigma}{d\Omega} = \frac{\rho}{128\pi^2 s}[|M_{++}|^2 + |M_{+-}|^2]$$

where $\rho = [1 - 4m_{pi}^2/s]$ and $s = M_{\pi\pi}^2$. The helicity amplitude M_{++}, M_{+-} (helicity difference $\lambda = 0, 2$ respectively) are partial-wave expanded as

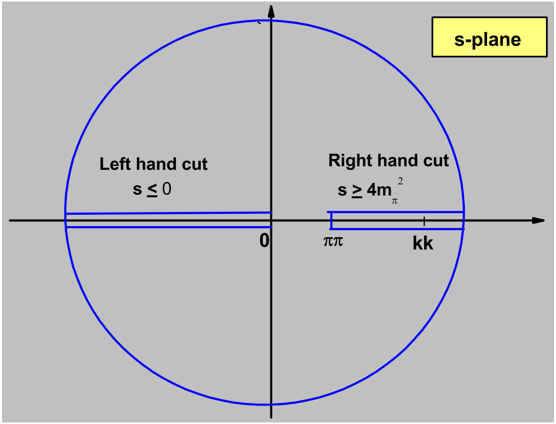
$$M_{++}(s,\theta,\phi) = e^2\sqrt{16}\sum_{J\geq 0} F_{J0}(s)Y_{J0}(\theta,\phi)$$

$$M_{+-}(s,\theta,\phi) = e^2\sqrt{16}\sum_{J\geq 2} F_{J2}(s)Y_{J2}(\theta,\phi)$$

Denoting the s-channel partial waves of the Born amplitydes by $B_{J\lambda}(s)$, the low energy theorem requires:

$$\frac{F_{J\lambda}(s)}{B_{J\lambda}(s)} \to 1, \qquad F_{J\lambda}(s) - B_{J\lambda}(s) = O(s), \quad s \to 0$$

Each partial wave, $F_{J\lambda}(s)$,has a right hand cut from $s = 4m_\pi^2$ to $+\infty$. and a left hand cut from $s = 0$ to $-\infty$.



Their asymptotics (from Regge behaviour ensures that the difference $(F_{J\lambda} - B_{J\lambda})/[s(s-4m_\pi^2)^{J/2}])$ satisfies an unsubtracted dispersion relation:

$$F_{J\lambda}(s) = B_{J\lambda}(s) + L_{J\lambda}(s) + \frac{s(s-4m_\pi^2)^{J/2}}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds' \, ImF_{J\lambda}(s')}{s'(s'-4m_\pi^2)^{J/2}(s'-s)}$$

where$(s-4m_\pi^2)^{J/2}$is the threshold behaviour of the Jth partial wave. $B_{J\lambda}$ is the partial wave projection of the OPE Born amplitude, while $L_{J\lambda}$ is that from other crossed-channel exchanges (ρ, ω, etc). To conform to the low energy theorem $L_{J\lambda}(s=0)=0$. B and L are specified by the left hand cut of the physical amplitude.

Unitarity requires that

$$ImF^{I}_{J\lambda} = \sum_{n} \rho_n F^{I}_{J\lambda}(\gamma\gamma \longrightarrow \ n) T^{I}_{J\lambda}{}^{*}(n \longrightarrow \pi\pi)$$

• $I = 0$ *Swave* :
Let us now consider the two-component vector formed with the amplitude $F_{\gamma\gamma\rightarrow\pi\pi}$and $F_{\gamma\gamma\rightarrow K\overline{K}}$

$$F^{I=0} = \begin{pmatrix} F^{I=0}_{\pi} \\ F^{I=0}_{K} \end{pmatrix}, \qquad \rho = \begin{pmatrix} \rho_\pi & 0 \\ 0 & \rho_K \end{pmatrix}$$

$$B^{I=0} = \begin{pmatrix} B^{I=0}_{\pi} \\ B^{I=0}_{K} \end{pmatrix}, \qquad T^{I=0,\dagger} = \begin{pmatrix} T^{I=0}_{11} & T^{I=0}_{12} \\ T^{I=0}_{21} & T^{I=0}_{22} \end{pmatrix}^{\dagger}$$

The input for the hadronic amplitudes is based on a modification (and extension) of the K-matrix parametrization of AMP(K.L.Au, D.Morgan, M.R.Pennington,Phys.Rev.D35(1987)1633) Briefly, the T-matrix is related to the K-matrix by $T = \frac{K}{1-i\rho K}$

To conform to the low energy theorem, we neglect $L_{J\lambda}$ at present. Use iteration approach:

$$F_1^{N+1} = B_1 + \frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda} \frac{\rho_\pi F_1^N(s^{'})T_{11}^*(s^{'})}{s^{'}(s^{'}-s)}ds^{'} + \frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda} \frac{\rho_K F_2^N T_{12}^*(s^{'})}{s^{'}(s^{'}-s)}ds^{'}$$

$$F_2^{N+1} = B_2 + \frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda} \frac{\rho_\pi F_1^N(s^{'})T_{21}^*(s^{'})}{s^{'}(s^{'}-s)}ds^{'} + \frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda} \frac{\rho_K F_2^N(s^{'})T_{22}^*}{s^{'}(s^{'}-s)}ds^{'}$$

- $I = 2$ *Swave* :

We input the $T_{\pi\pi}^{IJ=02}$ given by Z.Y.Zhou, G.Y.Qin, P.Zhang, Z.G.Xiao and H.Q.Zheng,JHEP.43(2005).
Use the iteration approach:

$$F_{IJ=20}^{N+1} = B_{\pi,IJ=20} + \frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda} \frac{\rho_\pi F_{IJ=20}^{N}(s^{'})T_{\pi\pi,IJ=20}^{*}(s^{'})}{s^{'}(s^{'}-s)}ds^{'}$$

- $I = 0$ *Dwave* $\lambda = 0, 2$:

For the $\gamma\gamma \to K^+K^-$ reaction the Born term contribution in J=2 is small compared to the one in S-wave up to about $\sqrt{s} = 1.4GeV$, due to the large K mass.This allows us to neglect this term to a good approximation in the range $\sqrt{s} < 1.4GeV$ where we are interest. An immediate consequence [K.M.Watson,PHYs.Rev.88 (1952)1163] is that in the region we considered the phase $\phi_{J\lambda}^{I}(s)$ of each $\gamma\gamma \to \pi\pi$ partial wave, $F_{J\lambda}^{I}(s)$, must equal the correspon-ding $\pi\pi \to \pi\pi$ phase shift $\delta_J^I(s) = argT_J^I(\pi\pi \to \pi\pi)$.

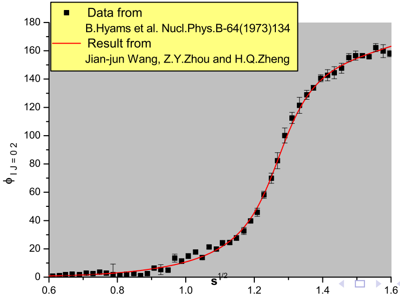
Consider the Omnès function

$$\Omega^I_{J\lambda}(s) = exp[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\phi^I_{J\lambda}(s')}{s'(s'-s)}ds']$$

Each spin amplitude can then be written as

$$F^I_{J\lambda}(s) = P^I_{J\lambda}(s)\Omega^I_{J\lambda}(s)$$

where Ω supplies the phase and concomitant amplitude variation, while P is real in the $\gamma\gamma \to \pi\pi$ physical region having only a left hand cut.

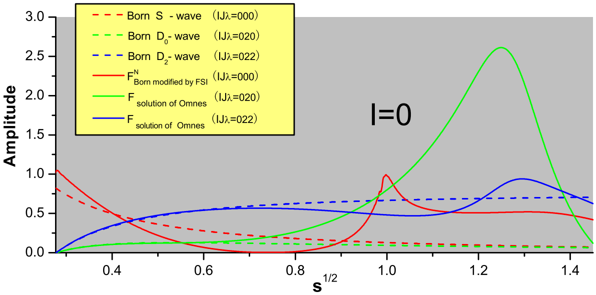


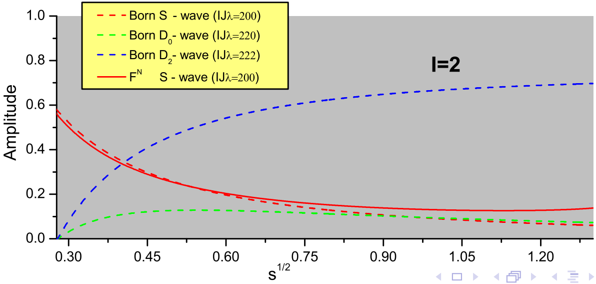
We input the $\phi^{I=0}_{J=2}$ from [Jian Jun Wang, Z.Y.Zhou and H.Q.Zheng (arxiv:hep-ph/0508040v2 8 Mar 2006)] and write down such forms of the equations to find the polynomials $P_1(s) and P_2(s)$:

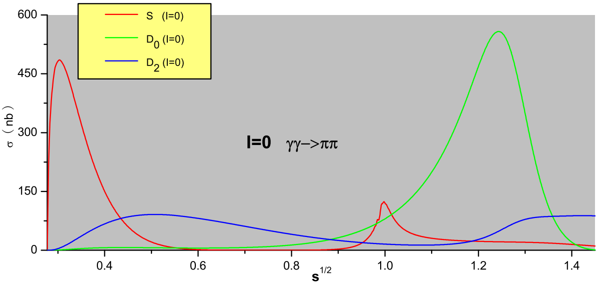
$$F^{I=0}_{J=2\lambda=0}(s) = B^{I=0}_{J=2\lambda=0}(s)P_1(s)\Omega^{I=0}_{J=2\lambda=0}(s)$$
$$F^{I=0}_{J=2\lambda=2}(s) = B^{I=0}_{J=2\lambda=2}(s)P_2(s)\Omega^{I=0}_{J=2\lambda=2}(s)$$
$$P_1(s) = (1 + a1s^2 + a2s^3 + a3^{s^4})$$
$$P_2(s) = (1 + b1s^2 + b2s^3 + b3s^4)$$

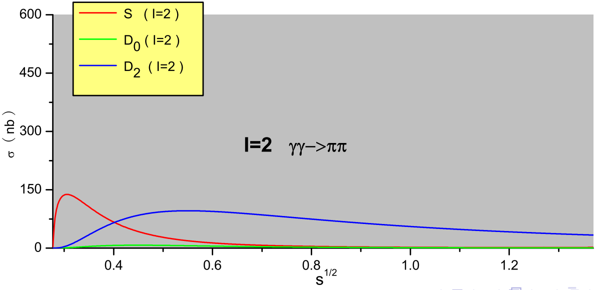
- $I = 2\ Dwave\ \lambda = 0, 2$:

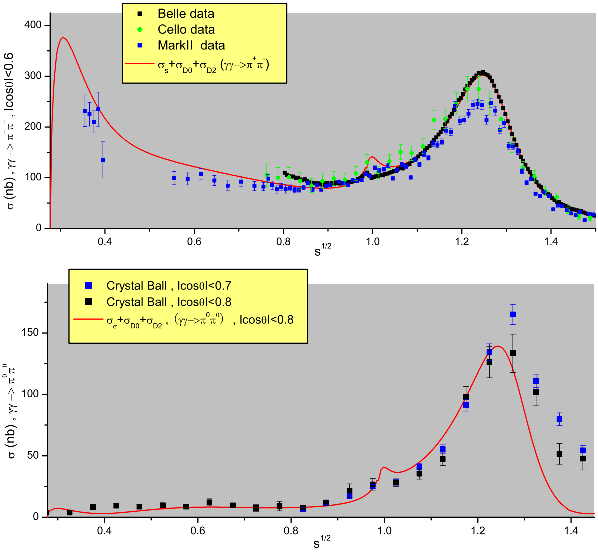
At present for these partial waves,we use the Born terms to a approximation in the range we are interested.


Born S - wave (IJλ=000)
Born D0- wave (IJλ=020)
Born D2- wave (IJλ=022)
F^N Born modified by FSI (IJλ=000)
F solution of Omnes (IJλ=020)
F solution of Omnes (IJλ=022)
I=0
Amplitude
s^1/2


Born S - wave (IJλ=200)
Born D0- wave (IJλ=220)
Born D2- wave (IJλ=222)
F^N S - wave (IJλ=200)
I=2
Amplitude
s^1/2


S (I=0)
D0 (I=0)
D2 (I=0)
I=0 γγ−>ππ
σ (nb)
600
450
300
150
0
0.4
0.6
0.8
1.0
1.2
1.4
s1/2


S (I=2)
D0 (I=2)
D2 (I=2)
I=2 γγ−>ππ
σ (nb)
600
450
300
150
0
0.4
0.6
0.8
1.0
1.2
s1/2


Belle data
Cello data
MarkII data
$\sigma_s+\sigma_{D0}+\sigma_{D2}$ $(\gamma\gamma\to\pi^+\pi^-)$
σ (nb), $\gamma\gamma\to\pi^+\pi^-$, |cosθ|<0.6
$s^{1/2}$
Crystal Ball , |cosθ|<0.7
Crystal Ball , |cosθ|<0.8
$\sigma_\sigma+\sigma_{D0}+\sigma_{D2}$, $(\gamma\gamma\to\pi^0\pi^0)$, |cosθ|<0.8
σ (nb), $\gamma\gamma\to\pi^0\pi^0$
$s^{1/2}$

We now can calculate the $\gamma\gamma$ couplings of the resonant states in the threshold to 1.4GeV region that our amplitude analysis determines. We can do this by two different methods. The first is based on the analytic continuation of the amplitudes we have found in our fit into the complex s-plane to the pole position. This is the only formally correct way of deducing the couplings of any resonance and its outcomes are free from background contamination. The second is a more naive approach based on the Breit-Wigner-like peak height. For the $f_0(980)$ only the pole method is applicable because of the overlapping of this state with $K\bar{K}$ threshold and the broader $f_0(400-1200)$. For this latter broad state, only the "peak height" provides a sensible measure of its $\gamma\gamma$ width, since its pole is too far from the real axis to be reliably located under the approximations needed to perform the analytic continuation.

To work out the pole residue based definition of the radiative widths, we suppose the strong interaction amplitudes $T_J(s)$ and the corresponding $\gamma\gamma \to \pi\pi$ amplitudes $F(s)$ to be dominated by their pole contribution near the resonance pole,i.e. for $s \sim s_R$; then we can write them in the form

$$T_J(\pi\pi \to \pi\pi)(s \sim s_R) = \frac{g_\pi^2}{s_R - s}$$
$$F_{J\lambda}(\gamma\gamma \to \pi\pi)(s \sim s_R) = \frac{g_\gamma g_\pi}{s_R - s}$$

The corresponding width is then evaluated using the formula

$$\Gamma^R_{\gamma\gamma}(pole) = \frac{\alpha^2 \beta_R |g_\gamma|^2}{4(2J+1)m_R}$$

where $\alpha = 1/137$ is the fine structure constant, and $\beta_R = (1 - 4m_\pi^2/m_R^2)^{1/2} \simeq 1$

Because of its very large width, the $f_0(400-1200)$ coupling to two photon cannot be calculated with this technique, since on cannot reliably continue so far into the complex plane. As an alternative, there is a rough estimate of its width by using an expression based on the standard resonance peak formula

$$\Gamma^R_{\gamma\gamma}(peak) = \frac{\sigma_{\gamma\gamma}(res.peak) m_R^2 \Gamma_{tot}}{8\pi(\hbar c)^2(2J+1)BR}$$

where BR is the hadronic branching ratio for the final state considered.

Thanks !