Radiative $B \to (K_1, b_1, h_1)$ decays at Next-to-Leading Order in LEET

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Aims To Study B-meson decays

- Determine the elements of CKM matrix and explore the origin of CP violation.
- Study the strong interaction dynamics related to the confinement of quarks and gluons inside hadrons.
- Explore the possibility of New Physics beyond the Standard Model in B-decays.

All these tasks complement each other.

WHY RARE DECAYS

- ✓ Rare B-decays ($b \to s\gamma, b \to d\gamma, b \to s\ell^+\ell^-, \ldots$) are Flavor-Changing-Neutral-Currents (FCNC) processes.
- ✓ In SM, all electrically neutral bosons have only Flavor-diagonal couplings. Hence, in the SM, FCNC processes are not allowed at Tree level.
- ✓ These decays play an important role in the determination of CKM matrix elements.
- ✓ FCNC processes are sensitive to physics beyond the SM, such as supersymmetry.
- ✓ Last, but not least, Rare B-decays enjoy great attention in the ongoing and planned experimental programmes in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

Effective Hamiltonian

$$H_{\mathrm{eff}} = G_F/2 \left\{ \begin{array}{l} V_{ub}V_{us}^{*}\left[C_1^{(u)}(\mu)O_1^{(u)}(\mu) + C_2^{(u)}(\mu)O_2^{(u)}(\mu) \right] + \\ V_{cb}V_{cs}^{*}\left[C_1^{(c)}(\mu)O_1^{(c)}(\mu) + C_2^{(c)}(\mu)O_2^{(c)}(\mu) \right] \\ -V_{tb}V_{ts}^{*}\left[C_7^{\mathrm{eff}}(\mu)O_7(\mu) + C_8^{\mathrm{eff}}(\mu)O_8(\mu) \right] + \cdots \end{array} \right\}$$

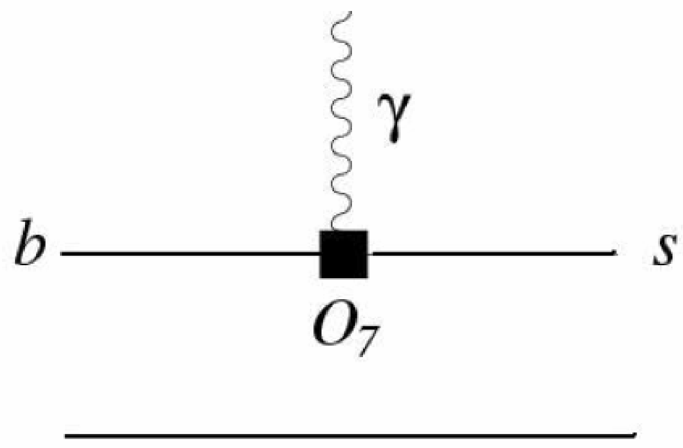
$$O_1^{p} = (\bar{s}_i p_j)_{V-A}(\bar{p}_j b_i)_{V-A}$$

$$O_2 = (\bar{s}_i p_i)_{V-A}(\bar{p}_j b_j)_{V-A}$$

$$O_7 = \frac{e}{4\pi^2}\bar{s}\sigma_{\mu\nu}(m_s P_L + m_b P_R) b F^{\mu\nu}$$

$$O_8 = \frac{g_s m_b}{8\pi^2}\bar{s}_i \sigma^{\mu\nu}(1+\gamma_5) T^a_{ij} b_j G^a_{\mu\nu}$$

Leading order $B \to K_1 \gamma$ Calculations



$$\langle O_7 \rangle_A \equiv \langle K_1(p', \epsilon)\gamma(q, e)|O_7|B(p)\rangle$$

$$= \frac{em_b}{4\pi^2}\xi^{(K_1)}\left[\epsilon^* \cdot q(p+p') \cdot e^* - \epsilon^* \cdot e^*(p^2 - p'^2) + i\epsilon_{\mu\nu\alpha\beta} e^{*\mu}\epsilon^{*\nu} q^\alpha (p+p')^\beta\right]$$

$$\Gamma(B \to K_1\gamma) = \frac{G_F^2 \alpha m_b^2 m_B^3}{32\pi^4}|V_{tb}V_{ts}^*|^2\left(1 - \frac{m^2}{m_B^2}\right)^3 |\xi^{(K_1)}|^2 |C_7^{\mathrm{eff}(0)}|^2$$

Why We Go at Next to leading order?

- The main motivation to do NLO calculation is that the theoretical uncertainties (like the scale dependence) get reduced at this order.
- The other issue is the residual renormalization scale dependence of the result which in principally not to be there if one measure the physical quantities.

But the cost we have to pay is in the form of complicated formulas which you will see soon.

There are two major contributions:

- Vertex Corrections (which have nothing to do with the out going meson)
- Hard Spectator Corrections (Depends on the process we are going to study)

Factorization Formula

The formula used to calculate these Next-to-leading order corrections is given by Beneke and Feldmann and it is:

$$f_i(q^2) = C_i \xi_{K_1} + \Phi_B \otimes T_i \otimes \Phi_{K_1}$$

KINEMATICS

$$p_b^\mu \approx m_b v^\mu, \qquad l^\mu = \frac{l_+}{2} n_+^\mu + l_\perp^\mu + \frac{l_-}{2} n_-^\mu$$

$$k_1^\mu \approx uEn_-^\mu + k_\perp^\mu + O(k_\perp^2),\; k_2^\mu \approx \bar{u}En_-^\mu - k_\perp^\mu + O(k_\perp^2)$$

$$v^\mu = (n_-^\mu + n_+^\mu)/2 \qquad E \approx \frac{M}{2}, \quad \omega \approx \frac{M}{2}$$

$$q^\mu = \omega n_+^\mu \qquad l_\pm \sim \Lambda_{QCD}$$

T_i : is the Hard Spectator corrections

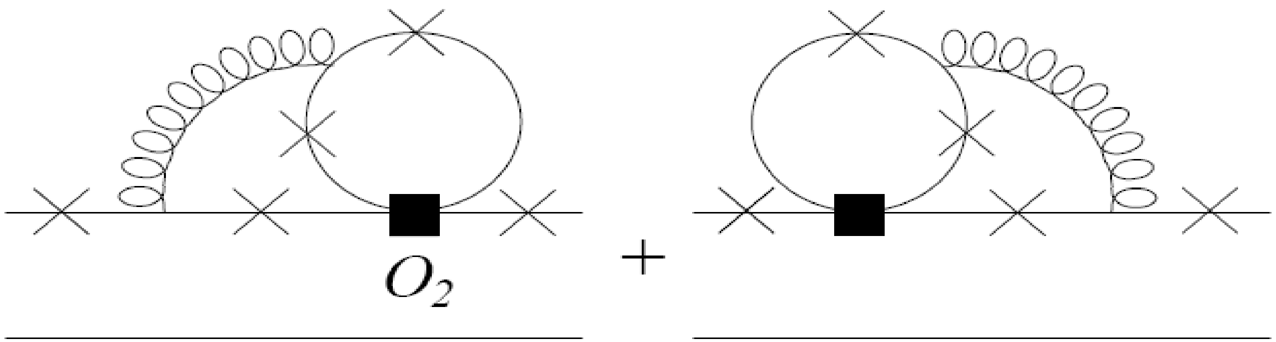
$$\Delta\mathcal{M}^{(\mathrm{HSA})} = \frac{4\pi\alpha_s C_F}{N_c}\int_0^1 du \int_0^\infty dl_+ \, M_{jk}^{(B)} M_{li}^{(K_1)} \mathcal{T}_{ijkl}$$

$M_{jk}^{(B)}$ and $M_{li}^{(K_1)}$ are B-meson and K₁ – meson projection operators

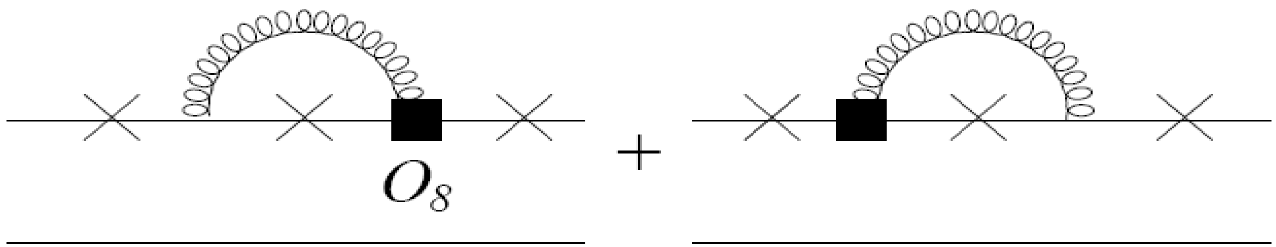
$$M_{jk}^{(B)} = -\frac{i f_B M}{4}\left[\frac{1+\not{v}}{2}\left\{\phi_+^{(B)}(l_+)\not{n}_+ + \phi_-^{(B)}(l_+)\times\left(\not{n}_- - l_+\gamma_\perp^\mu \frac{\partial}{\partial l_\perp^\mu}\right)\right\}\gamma_5\right]_{jk}\Bigg|_{l=(l_+/2)n_+}$$

$$M_{li}^{(K_1)} = -\frac{i}{4}\left[f_\perp^{(K_1)}\not{\varepsilon}^*\not{p}\phi_\perp^{(K_1)}(u) + f_{//}^{(K_1)}\not{p}\frac{m}{E}(v\varepsilon^*)\phi_{//}^{(K_1)}(u)\right]$$

Hard Vertex Corrections



(a)



(b)

Results of these Vertex Corrections

$$\langle O_2 \rangle_{VC} = \frac{\alpha_s}{4\pi} \langle O_7 \rangle \left(\frac{416}{81} \ln \frac{m_b}{\mu} + r_2 \right)$$

$$\langle O_8 \rangle_{VC} = \frac{\alpha_s}{4\pi} \langle O_7 \rangle \left[-\frac{32}{9} \ln \frac{m_b}{\mu} + \frac{4}{27} (33 - 2\pi^2 + 6i\pi) \right]$$

$$r_2 = \frac{2}{243} \{ -833 + 144\pi^2 z^{3/2}$$

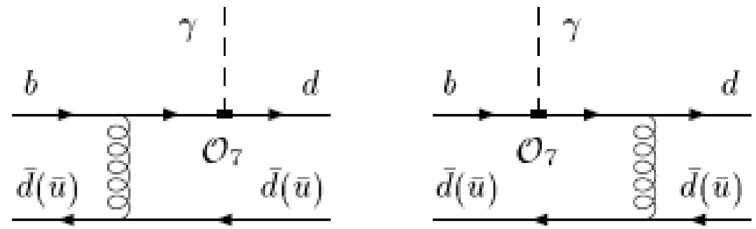
$$+ \left[\begin{array}{c} 1728 - 180\pi^2 - 1296\zeta(3) + \\ (1296 - 324\pi^2)L + 108L^2 + 36L^3 \end{array} \right] z$$

$$+ \begin{array}{c} [648 + 72\pi^2 + (432 - 216\pi^2)L + 36L^3]z^2 \\ +[-54 - 84\pi^2 + 1092L - 756L^2] \end{array} z^3 \}$$

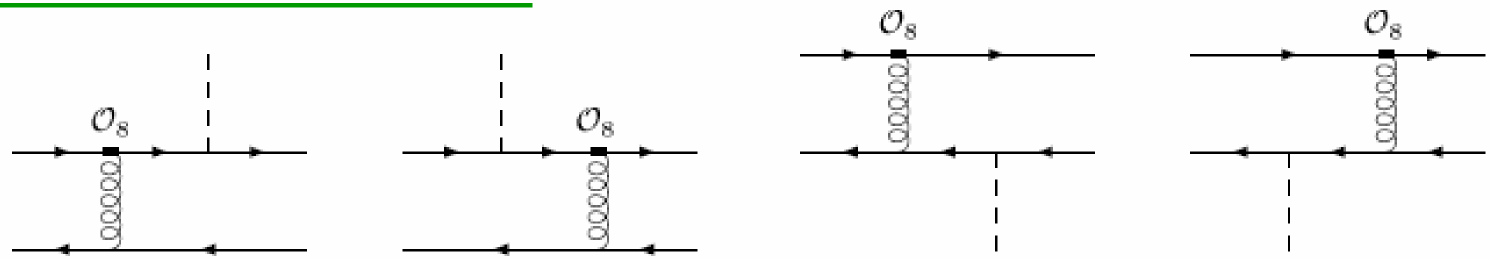
$$+ i\frac{16\pi}{81} \{ -5 + \begin{array}{c} [45 - 3\pi^2 + 9L + 9L^2]z + \\ [-3\pi^2 + 9L^2]z^2 + [28 - 12L]z^3 \end{array} \}$$

Hard Spectator contributions

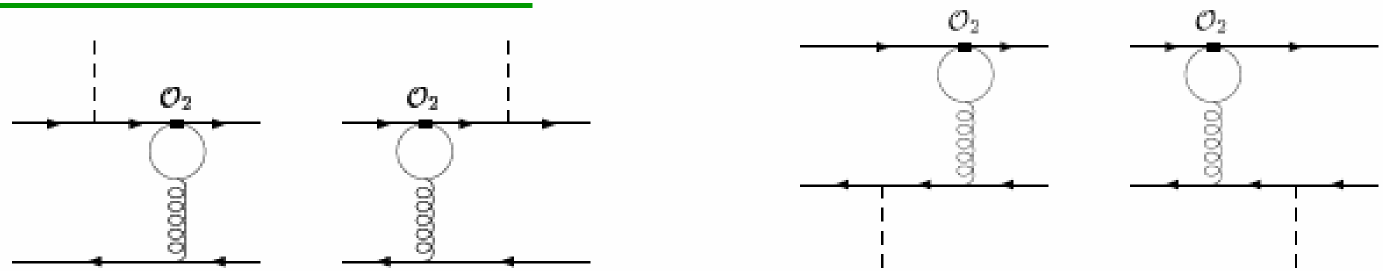
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



Branching Ratio

$$\mathcal{B}_{\rm th}(B\to K_1\gamma) = \tau_B \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K_1)}\right]^2 \left(1-\frac{m_{K^*}^2}{M^2}\right)^3 \left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2$$

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)K_1}(\mu_{\rm sp})$$

$$A_{C_7}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi} C_7^{(1)\rm eff}(\mu),$$

$$A_{\rm ver}^{(1)}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left\{\frac{32}{81}\left[13C_2^{(0)}(\mu) + 27C_7^{(0)\rm eff}(\mu) - 9C_8^{(0)\rm eff}(\mu)\right]\ln\frac{m_b}{\mu}\right.$$

$$\left. - \frac{20}{3}C_7^{(0)\rm eff}(\mu) + \frac{4}{27}\left(33-2\pi^2+6\pi i\right)C_8^{(0)\rm eff}(\mu) + r_2(z)C_2^{(0)}(\mu)\right\},$$

$$A_{\rm sp}^{(1)K_1}(\mu_{\rm sp}) = \frac{\alpha_s(\mu_{\rm sp})}{4\pi}\frac{2\Delta F_\perp^{(K_1)}(\mu_{\rm sp})}{9\xi_\perp^{(K_1)}}\left\{3C_7^{(0)\rm eff}(\mu_{\rm sp})\right.$$

$$\left. + C_8^{(0)\rm eff}(\mu_{\rm sp})\left[1-\frac{6a_{\perp 1}^{(K_1)}(\mu_{\rm sp})}{\langle \bar u^{-1}\rangle_\perp^{(K_1)}(\mu_{\rm sp})}\right] + C_2^{(0)}(\mu_{\rm sp})\left[1-\frac{h^{(K_1)}(z,\mu_{\rm sp})}{\langle \bar u^{-1}\rangle_\perp^{(K_1)}(\mu_{\rm sp})}\right]\right\}$$

- $z = m_c^2/m_b^2$; $\mu_{\rm sp} = \sqrt{\mu\Lambda_H}$; $\Lambda_H = O(\Lambda_{\rm QCD})$

m_c/m_b	0.29	0.29	0.29
μ	$\bar{m}_b = 4.27\text{GeV}$	$m_{b,\text{pole}} = 4.65\text{GeV}$	$m_{b,\text{PS}} = 4.6\text{GeV}$
$\left(C_7^{(0)\text{eff}} + A^{(1)}(\mu)\right)_{\text{Our}}$	$-0.358 - i0.022$	$-0.356 - i0.021$	$-0.356 - i0.021$
$\left(C_7^{(0)\text{eff}} + A^{(1)}(\mu)\right)_{\text{Lee}}$	$-0.406 - i0.033$	$\times$	$-0.410 - i0.033$
$\left\|C_7^{(0)\text{eff}} + A^{(1)}(\mu)\right\|^2_{\text{Our}}$	0.128	0.127	0.127

$$\mathcal{B}(B^+ \to K_1^+\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$$

From the value of the branching ratio given above one can easily construct the value of only unknown, i.e. the form factor and the value is:

$$\xi_\perp^{(K_1)}(0) = 0.32 \pm 0.03$$

CP-Asymmetry

We also recall that the operator basis in $\mathcal{H}_{eff}$ is larger than what is shown in which the operators multiplying the CKM factor $V_{ub}V_{us}^*$ have been neglected. To calculate CP-asymmetry we have to put them back. Doing this, and using the unitarity relation $V_{cb}V_{cs}^* = -V_{ub}V_{us}^* - V_{tb}V_{ts}^*$,the effective Hamiltonian reads:

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} \left\{ \begin{array}{l} V_{tb}V_{ts}^*[C_7(\mu)O_7(\mu) + C_8(\mu)O_8(\mu) + C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] \\ V_{ub}V_{us}^*[C_1(\mu)(O_{1u}(\mu) - O_1(\mu)) + C_2(\mu)(O_{2u}(\mu) - O_2(\mu)) + \ldots] \end{array} \right\}$$

$$O_{1u}(\mu) = (\bar{s}_L\gamma_\mu T^a u_L)(\bar{u}_L\gamma^\mu T_a b_L)$$

$$O_{2u}(\mu) = (\bar{s}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu b_L)$$

Considering the lowest order result of annihilation amplitude, one has:

$$\mathcal{B}_{\rm th}(B^\pm \to K_1^\pm\gamma) = \tau_{B^+}\Gamma_{\rm th}(B^\pm \to K_1^\pm\gamma)$$

$$= \tau_{B^+}\frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left(1-\frac{m_{K_1}^2}{M^2}\right)^3 \left[\xi_\perp^{(K_1)}(0)\right]^2$$

$$\times\Big\{(C_7^{(0)\rm eff}+A_R^{(1)})^2+(F_1^2+F_2^2)(A_R^u+L_R^u)^2$$

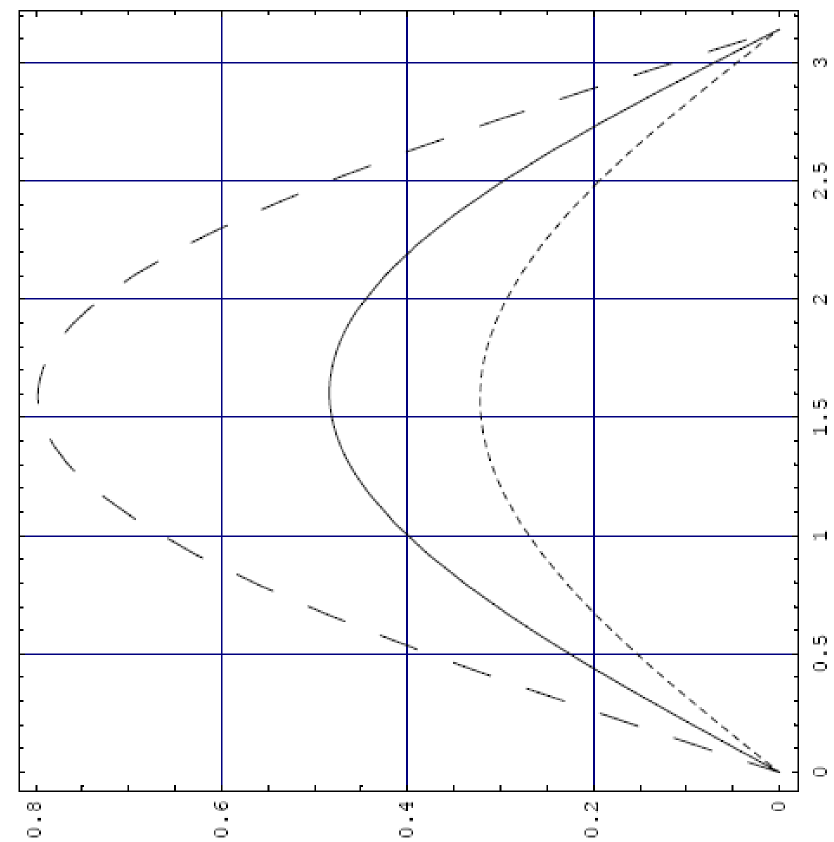
$$+2F_1[C_7^{(0)\rm eff}(A_R^u+L_R^u)+A_R^{(1)}L_R^u]\mp 2F_2[C_7^{(0)\rm eff}A_I^u-A_I^{(1)}L_R^u]\Big\}$$

Where

$$A^u(\mu)=\frac{\alpha_s(\mu)}{4\pi}C_2^{(0)}(\mu)[r_2(z)-r_2(0)]-\frac{\alpha_s(\mu_{\rm sp})}{18\pi}C_2^{(0)}(\mu_{\rm sp})\frac{\Delta F_\perp^{(K_1)}(\mu_{\rm sp})}{\xi_\perp^{(K_1)}(0)}\frac{h^{(K_1)}(z,\mu_{\rm sp})}{\langle\bar{u}^{-1}\rangle_\perp^{(K_1)}(\mu_{\rm sp})}$$

$$\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}=-\left|\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}\right|e^{i\alpha}=F_1+iF_2 \qquad L_R^u=\epsilon_A C_7^{(0)\rm eff}$$

$$\mathcal{A}_{CP}(K_1^\pm\gamma)=\frac{2F_2\left(A_I^u-\epsilon_A A_I^{(1)}\right)}{C_7^{(0)\rm eff}\left(1+2\epsilon_A\left[F_1+\frac{1}{2}\epsilon_A\left(F_1^2+F_2^2\right)\right]\right)}$$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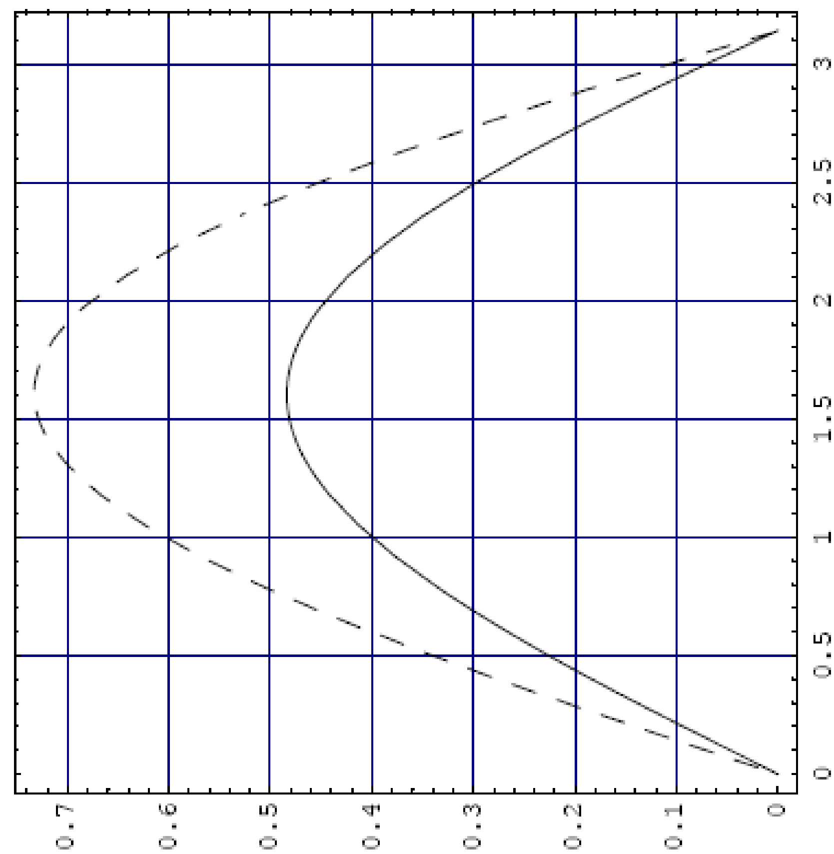
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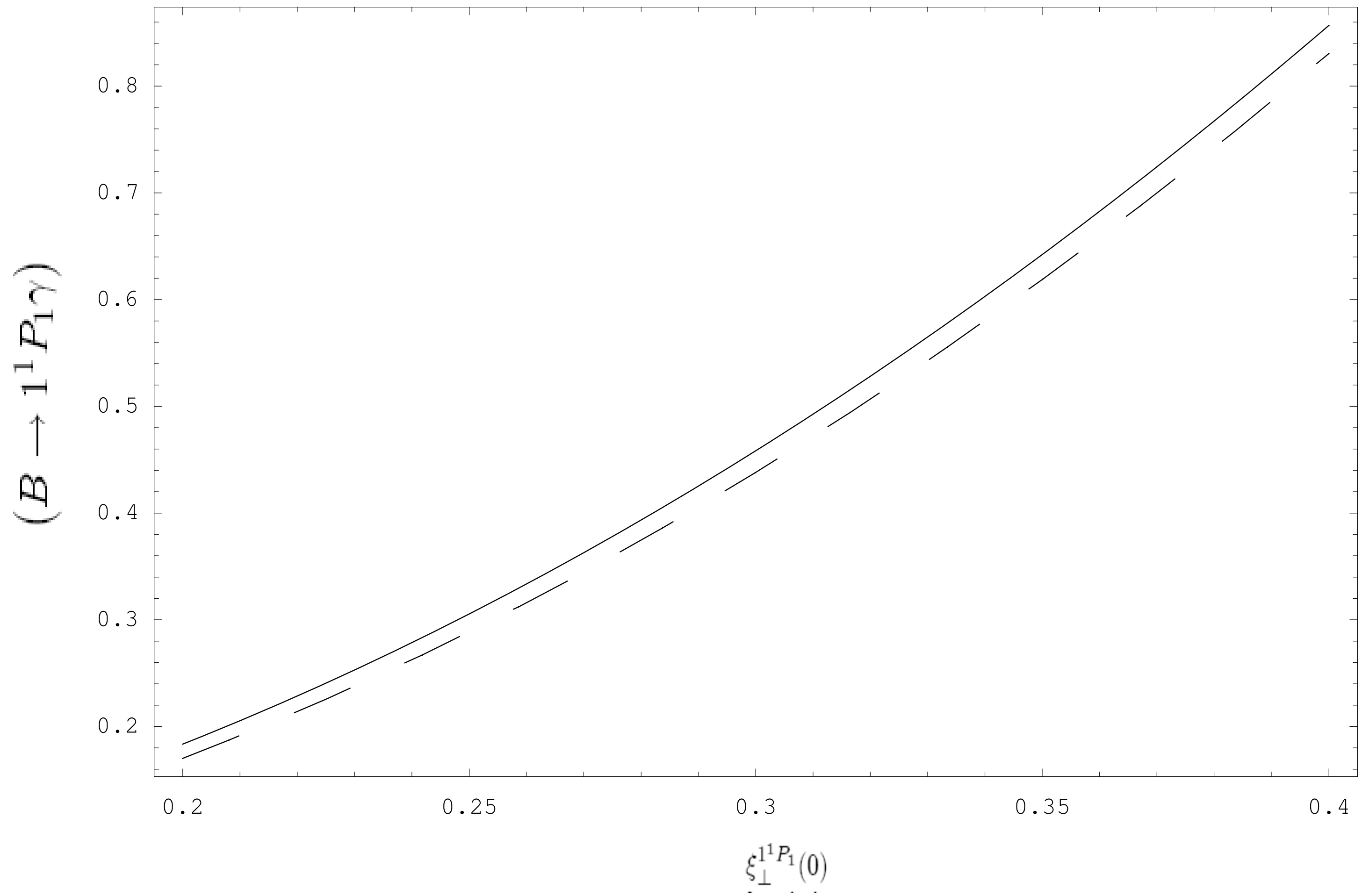
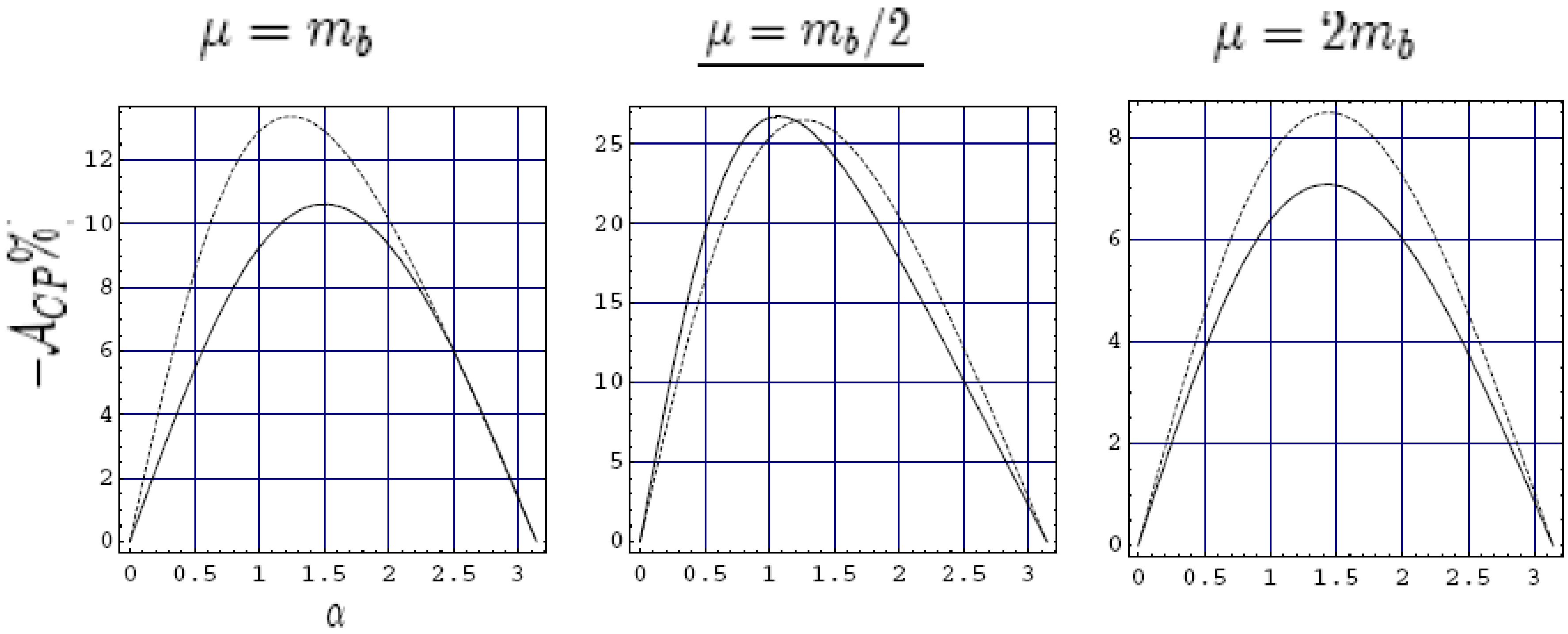
Branching Ratio and CP-asymmetry for $B \to 1^1P_1\ \gamma$ decays



CP-Asymmetry for these Decays



(a) (b) (c)

Conclusion-II

- We have Studied complete NLO calculations for $B \to K_1 \gamma$ decays.
- Hard Vertex and Hard Spectator corrections to the matrix elements are the two contributions that come at NLO which are discussed with some details.
- The value of form factor, the only unknown parameter in the calculation, has been extracted from the experimental value of the decay rate.
- CP-Asymmetry has been studied for these decays and it is found that the Hard Spectator corrections reduces it value from the vertex corrections alone. Then the sensitivity of the measured CP value with the scale has also been discussed.
- Following the same lines Branching ratio and CP – Asymmetry for $B \to (b_1, h_1)\gamma$ decays have been discussed.

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Thanks