On nonperturbative renormalization

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Introduction

- Chiral EFT approach to NN interactions (nonperturbative). Weinberg's proposal: $T = V + VG_0T$; V constructed from chiral perturbation theory. Advent of EFT approach to nuclear systems, basing on guiding principles from QCD—chiral symmetry;
- This approach proves successful, but there remain some difficult issues or challenges. One of the most persevering ones: Disputes over the EFT power counting and renormalization. Origin: The presence of large scattering lengths (hence unnatural from the EFT perspective) in low energy NN scattering poses serious challenges to treating power counting for EFT interactions *and* renormalization coherently in nonperturbative regime.
- Recent evidences (Phys. Rev. C72 (2005) 054006, Phys. Rev. C77 (2008) 014002, arXiv: 0709.2770[nucl-th]) show that a few counter terms 'suffice' in nonperturbative regime (at least for 1S0 and other channels) with potentials truncated up to N3LO. That means perturbative arguments or concepts about counterterms and anomalous dimension of EFT operators may be misleading in nonperturbative regimes (Kaplan, arXiv: nucl-th/0510023).

Reanalysis: (I) Rigorous solutions of LSE in pionless EFT

- Pionless EFT and factorized LSE for NN scattering in partial wave formalism (Ann. Phys. 263 (1998) 255, Phys. Rev. C71 (2005) 034001) (Consider uncoupled channels for simplicity):

$$V(q,q') = \sum_{i,j} C_{ij} q^{2i} q'^{2j} = U(q)\tilde{C}U^T(q'),$$

$$T(q,q') = \sum_{i,j} \tau_{ij} q^{2i} q'^{2j} = U(q)\tilde{\tau}U^T(q'),$$

$$U(q) = \begin{pmatrix} 1 & q^2 & q^4 & \cdots \end{pmatrix} \qquad (1)$$

Then Lippmann-Schwinger equation (LSE) reduces to the following algebraic form:

$$\tilde{\tau} = \tilde{C} + \tilde{C}\hat{I}\tilde{\tau}, \qquad \hat{I} = \begin{pmatrix} I_{00} & I_{01} & \cdots \\ I_{10} & I_{11} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \qquad (2)$$

where $I_{nl} = \int \frac{d^3k}{(2\pi)^3} \frac{k^{2(n+l)}}{E - k^2/M + i\varepsilon}$ are ill-defined integrals and M is nucleon mass.

- (Nonperturbative) renormalization: to render the integrals (I_{nl}) finite. These integrals could be generally parametrized as follows: $I_{nl} = \sum_{m=1}^{n+l} J_{2m+1} p^{2(n+l-m)} - J_0 p^{2(n+l)} - i\frac{Mp}{4\pi} p^{2(n+l)}$, with $J_{\ldots}$ being finite in a renormalization prescription. J_0 is singled out for reasons to be clear shortly. (Phys. Rev. C71 (2005) 034001)

- Rigorous on-shell T matrix($p=\sqrt{ME}$):

$$\frac{1}{T}=J_0+i\frac{Mp}{4\pi}+\frac{N(C_{\dots},J_{\dots},p^2)}{D(C_{\dots},J_{\dots},p^2)}, \qquad (3)$$

where $N=N_0+N_1p^2+\cdots$, $D=p^{2L}(D_0+D_1p^2+\cdots)$, and $N_j,D_{j'}$ are also polynomials in terms of the EFT couplings $C_{\dots}$ and renormalization parameters $J_{\dots}$ (except J_0), i.e., they are formally 'chiral' perturbative. Note, J_0 'stands' alone in the inverse T, unlike the rest of $J_{\dots}$ that are 'entangled' with EFT couplings in the rigorous solutions of T.

- Examples (Notation: in $O\left(\frac{Q}{\Lambda}\right)^{\Delta}$, Δ is chiral order, Q is an EFT scale, Λ is the upper scale in EFT expansion)

(1) 1S0 channel：

$$\Delta=0,\qquad V=C_0,\quad \frac{1}{T}=J_0+i\frac{Mp}{4\pi}+\frac{1}{C_0}; \qquad (4)$$

$$\Delta=2,\qquad V=C_0+C_1(q^2+q'^2),\quad \frac{1}{T}=J_0+i\frac{Mp}{4\pi}+\frac{N}{D},\quad N=(1-C_1J_3)^2,$$

$$D=C_0+C_1^2J_5+C_1(2-C_1J_3)p^2; \qquad (5)$$

$$\Delta=4,\qquad V=C_0+C_1(q^2+q'^2)+C_2(q^4+q'^4)+\tilde{C}_2q^2q'^2,\quad \frac{1}{T}=J_0+i\frac{Mp}{4\pi}+\frac{N}{D},$$

$$N=N_0+N_1p^2+N_2p^4,\quad D=D_0+D_1p^2+D_2p^4+D_3p^6,$$

$$\begin{aligned}N_0&=(1-C_1J_3-C_2J_5)^2-\tilde{C}_2J_5-C_0\tilde{C}_2J_3^2+2C_2\tilde{C}_2(J_5^2-J_3J_7)\\&\quad-C_2^2\tilde{C}_2(J_5^3+J_3^2J_9-2J_3J_5J_7)\end{aligned},$$

$$N_1=-2C_2J_3-\tilde{C}_2J_3+2C_1C_2J_3^2+2C_2(\tilde{C}_2+C_2)J_3J_5+C_2^2\tilde{C}_2(J_3^2J_7-J_3J_5^2),$$

$$N_2=C_2^2J_3^2,$$

$$D_0=C_0(1-\tilde{C}_2J_5)+C_1^2J_5+C_2^2J_9+2C_1C_2J_7+C_2^2\tilde{C}_2(J_7^2-J_5J_9),$$

$$D_1 = C_1(2 - C_1 J_3) + C_0 C_2 J_3 + C_2(C_2 + 2\tilde{C}_2) J_7 + C_2^2 \tilde{C}_2 (J_3 J_9 - J_5 J_7),$$

$$D_2 = 2C_4(1 - C_1 J_3) + \tilde{C}_2(1 - C_2 J_5)^2 - C_2^2 J_5 - C_2^2 \tilde{C}_2 J_3 J_7,$$

$$D_3 = -C_2^2 J_3. \tag{6}$$

(2) P channels：

$$\Delta = 2, \quad V = C_{P;0} q q', \quad \frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \left(\frac{1}{C_{P;0}} - J_3\right) p^{-2}; \tag{7}$$

$$\Delta = 4, \quad V = C_{P;0} q q' + C_{P;1} q q' (q^2 + q'^2), \quad \frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{N}{D} p^{-2},$$

$$N = (1 - C_{P;1} J_5)^2 - C_{P;0} J_3 - C_{P;1}^2 J_3 J_7 - C_{P;1} J_3 (2 - C_{P;1} J_5) p^2,$$

$$D = C_{P;0} + C_{P;1}^2 J_7 + C_{P;1}(2 - C_{P;1} J_5) p^2; \tag{8}$$

$\Delta = 6$, ……

(3) D channels

$$\Delta = 4, \quad V = C_{D;0} q^2 q'^2, \quad \frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \left(\frac{1}{C_{D;0}} - J_3 p^2 - J_5\right) p^{-4}; \tag{9}$$

$$\Delta = 6, \quad V = C_{D;0} q^2 q'^2 + C_{D;1} q^2 q'^2 (q^2 + q'^2), \quad \frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{N}{D} p^{-4},$$

$$N = (1 - C_{D;1} J_7)^2 - C_{D;0} J_5 - C_{D;1}^2 J_5 J_9 - \left(C_{D;0} J_3 + C_{D;1}\left(2J_5 + C_{D;1}(J_3 J_9 - J_5 J_7)\right)\right) p^2$$
$$- C_{D;1} J_3 (2 - C_{D;1} J_7) p^4$$

$$, \quad D = C_{D;0} + C_{D;1}^2 J_9 + C_{D;1}(2 - C_{D;1} J_7) p^2; \tag{10}$$

$\Delta = 8$, ……

*Note that in all the above channels, we have $\frac{Dp^{2L}}{N}|_{J_{\dots}=0} = V|_{on-shell}$.

*Warning: (1) Nonperturbative 'subtraction' is necessary (on integrals rather than on vertices, Phys. Lett. B429 (1998) 227). (2) Different regularization schemes yield inequivalent results before subtraction is performed in nonperturbative regime(Ann. Phys. 263 (1998) 255). E.g., for 1S0 in DR, $J_{\cdots} = 0$, then, with $\varepsilon \to 0$, we have

$$\Delta = 0, \quad \frac{1}{T} = i\frac{Mp}{4\pi} + \frac{1}{C_0};$$

$$\Delta = 2, \quad \frac{1}{T} = i\frac{Mp}{4\pi} + \frac{1}{C_0 + 2C_1 p^2};$$

$$\Delta = 4, \quad \frac{1}{T} = i\frac{Mp}{4\pi} + \frac{1}{C_0 + 2C_1 p^2 + (2C_2 + \tilde{C}_2)p^4},$$

While in cutoff scheme with $\Lambda \to \infty$, we have

$$\Delta = 0, \quad \frac{1}{T} = \frac{M\Lambda}{4\pi} + i\frac{Mp}{4\pi} + \frac{1}{C_0};$$

$$\Delta = 2, \quad \frac{1}{T} = \frac{M\Lambda}{4\pi} + i\frac{Mp}{4\pi} + \frac{(1 - C_1\frac{M\Lambda^3}{4\pi})^2}{C_0 + C_1^2\frac{M\Lambda^5}{4\pi} + C_1(2 - C_1\frac{M\Lambda^3}{4\pi})p^2};$$

$\Delta = 4, \quad \ldots\ldots$

Hard to obtain equivalent T matrices across the two regularization schemes in nonperturbative regime.

Reanalysis: (II) Novel features revealed from the above rigorous solution

From above we could find that, *in nonperturbative regime*:

(1) Only finite many divergent integrals or renormalization parameters are involved in the rigorous nonperturbative solutions of T matrices, not ‘infinite’ (from the iterative point of view of LSE), a novel notion of ‘finiteness’. That means, to obtain a finite T matrix, only a finite number of divergences are to be treated or ‘subtracted’ nonperturbatively. Recent progresses are in accordance with this notion.

(2) It is hard to devise a systematic program of subtraction (like the well known perturbative renormalization) on the nonperturbative T matrix in order to obtain a finite T matrix from the nonperturbative form beset with divergences, at least not generally established yet;

(3) J_0 holds a special position (except 1S0 at LO!). In fact for any L-wave, the p-dependence of the inverse on-shell T matrix $\frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{N_0 + N_1 p^2 + \cdots}{D_0 + D_1 p^2 + \cdots} p^{-4}$ is physical. That means as the appropriate ratios of the coefficients of this rational function of p are physical objects that are must be invariant as renormalization prescription or scale varies. Thus, J_0 must be a physical scale or

RG invariant in order to yield physical p-dependence in $\frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{N(p)}{D(p)} p^{-4}$ in all channels. So are the ratios like $N_j / N_0, D_j / N_0$.

(4) Strong prescription dependence, this is obvious from the form of the T matrix given above;

(5) Failure of some perturbative wisdoms about renormalization. Let us elaborate on this point:

In 1S0 at LO of potential, there is no further p-dependence besides the imaginary part. Here the scattering length could be obtained as $a = \frac{M}{4\pi \,\mathrm{Re}(1/T)} |_{p=0} = \frac{1}{J_0 + 1/C_0}$, where it is obvious that the $J_0 + 1/C_0$ must be RG invariant: $J_0 + 1/C_0 = \frac{M\mu}{4\pi} + 1/C_{0;R}(\mu)$ which allows us to define a renormalized coupling $C_{0;R}$ in similar fashion as in perturbative program: $C_{0;R}(\mu) = \frac{4\pi}{M(1/a - \mu)}$. This what Kaplan, Savage and Wise did in their approach (Phys. Lett. B424, 390 (1998)). However, this scenario breaks down as one goes one order higher about the potential, where J_0 alone must be RG invariant. At $\Delta = 2$ for 1S0, we could still define renormalized couplings for C_0 and C_1 via the requirement that the ratios $D_0 / N_0, D_1 / N_0$ must be RG invariant: $\frac{(1 - C_1 J_3)^2}{C_0 + C_1^2 J_5} = \frac{\left(1 - C_{1;R}(\mu) \frac{q_3 M \mu^3}{4\pi}\right)^2}{C_{0;R}(\mu) + C_{1,R}^2(\mu) \frac{q_5 M \mu^5}{4\pi}} \equiv \frac{1}{c_0}$,

$$\frac{(1-C_1 J_3)^2}{C_1(2-C_1 J_3)} = \frac{\left(1-C_{1;R}(\mu)\frac{q_3 M\mu^3}{4\pi}\right)^2}{C_{1;R}(\mu)\left(2-C_{1;R}(\mu)\frac{q_3 M\mu^3}{4\pi}\right)} \equiv \frac{1}{c_1}$$. The reason is that $N = N_0$ is independent of p!. Then we could obtain the following nonperturbative running couplings:

$$C_{0;R}(\mu) = \frac{c_0}{1+q_3 c_1 \frac{M\mu^3}{4\pi}} - \frac{4\pi q_5}{q_3^2 M\mu}\left(1-\sqrt{1+q_3 c_1 \frac{M\mu^3}{4\pi}}\right)^2,$$

$$C_{1;R}(\mu) = \frac{4\pi}{q_3 M\mu^3}\left(1-\sqrt{1+q_3 c_1 \frac{M\mu^3}{4\pi}}\right).$$ With such parameters, we could rewrite the on-shell T matrix of 1S0 at this order as $\frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{1}{c_0 + c_1 p^2}$. Interestingly, such nonperturbative 'running' couplings possess both IR and UV fixed points:

UV FP: $C_{0;R} \Rightarrow 0, C_{1;R} \Rightarrow 0$;

IR FP: $C_{0;R} \Rightarrow c_0, C_{1;R} \Rightarrow \frac{c_1}{2}$.

However, at higher order with $\Delta \geq 4$, the factor N in $\frac{1}{T} = J_0 + i\frac{Mp}{4\pi} + \frac{N}{D}$ become p-dependent, there will be extra constraints from the RG invariance of the ratios N_j / N_0 in addition to D_j / N_0. Then it is more difficult for the couplings and the running scale to conspire to fulfill all the requirements. In fact, since the same $J_{\ldots}$ should be used in all channels, each channel would pose different constraints upon the couplings and $J_{\ldots}$, making things even worse.

One way out is to abandon the conventional wisdom developed in perturbative regimes and seek for true nonperturbative understandings of the issues. That means novel notions and theoretical mechanisms are needed to tackle renormalization in nonperturbative regimes.

(In practice, these difficulties have been circumvented in most literature by turning to numerical approaches where a finite cutoff are employed, which is then no longer a true renormalization at all.)

(6) The above difficulties also imply that the most crucial work in nonperturbative regimes is find efficient methods to parametrize the nonperturbative divergence and their renormalization (or removal) in a general way. It is then possible to discuss other issues.

Reanalysis: (III) Nonperturbative scenario and predictions from pionless EFT

Here, within the realm of pionless EFT, we present a very simple scenario of power counting of both the EFT couplings and the renormalization prescription or the parameters $J_{\dots}$ in the nonperturbative regime, then we show some interesting predictions (arXiv: 0711.4637[nucl-th]):

$$|C_{L;n}| \propto \frac{4\pi}{M\Lambda^{2L+2n+1}}, \quad |J_0| \propto \frac{M\Lambda}{4\pi}, |J_{2m+1}| \propto \frac{M\mu^{2m+1}}{4\pi}, \tag{11}$$

Here the scale μ is of the order of a typical EFT scale. That is, the EFT couplings still follow the conventional EFT power counting (Weinberg), we refrain us from making any modification of it. In the meantime, most of the renormalization parameters scale similarly as the conventional ones except J_0 which holds a very special position as shown above.

Now, with such a nonperturbative scenario in the realm of contact potentials (pionless EFT), we could obtain some interesting theoretical predictions for the effective range parameters for the NN scattering defined as below, $\mathrm{Re}(-\frac{4\pi}{MT}p^{2L})|_{p\to 0} = -\frac{1}{a} + \frac{1}{2}r_e p^2 + \sum_{k=2}^{\infty} v_k p^{2k}$.

Using the rigorous solutions given above, we could find that some

ERE parameters are unnaturally sized while the rest are naturally sized. Here by naturalness we mean that, ERE are functions of the upper scale Λ, and unnaturalness refers to that of the smaller scale μ. The predictions for the uncoupled channels as summarized in the following table:

Channels	Natural ERE parameters	Unnatural ERE parameters
1S0	$\{r_e, v_k, k \geq 2\}$	$\{a\}$
1P1,3P0,3P1	$\{a, v_k, k \geq 2\}$	$\{r_e\}$
1D2,3D2	$\{a, r_e, v_k, k \geq 3\}$	$\{v_2\}$
L-waves	$\{a, r_e, v_k, k \geq 2, k \neq L, L \geq 3\}$	$\{v_L\}$

Similar predictions for the ERE parameters or scattering behaviors could be found in any nonrelativistic systems whose dynamics is governed by similar contact potentials and power counting considered here.

We stress that this is only true in the scenario given above, (11)!

For coupled channels like 3S1-3D1, etc., there are some modest modifications to the picture listed above due to the effects of mixing of partial waves.

Reanalysis: (IV) Approximate methods and test of EFT approach-Padé approximant

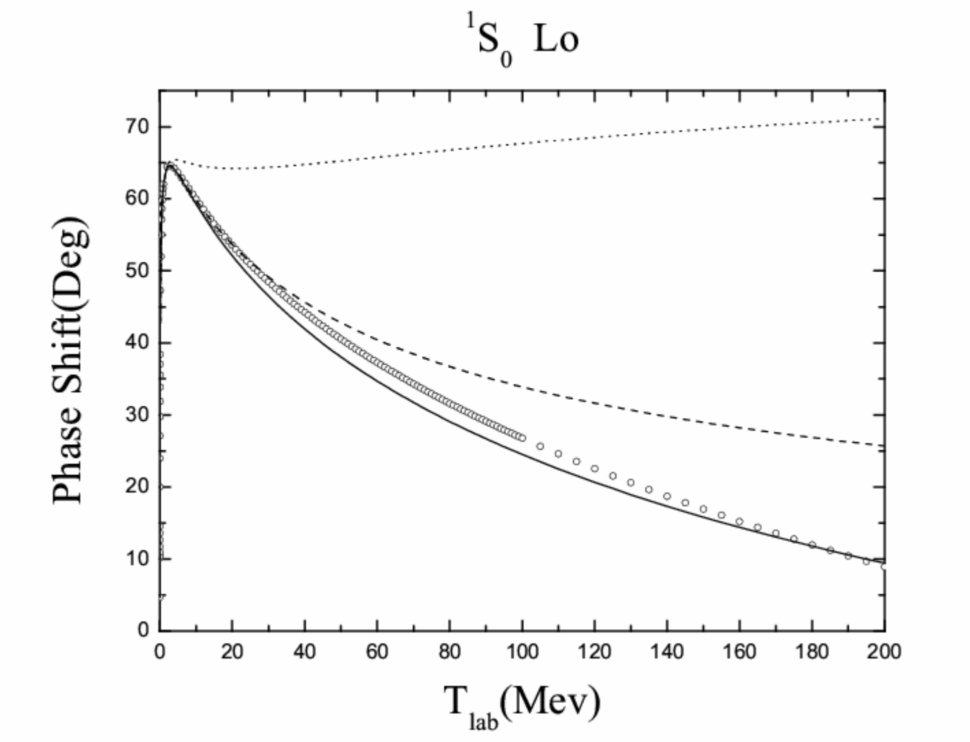
The above discussions are only valid within contact potentials though rigorous, as the realistic pion-exchange potentials are put aside. The dynamical pictures depicted above may be relevant to very low energy region NN scattering ($E << m_\pi$) where all interactions might be described by contact potentials, or pionless EFT. For modest energies ($E \approx m_\pi$), the pion-exchange potentials must be included explicitly.

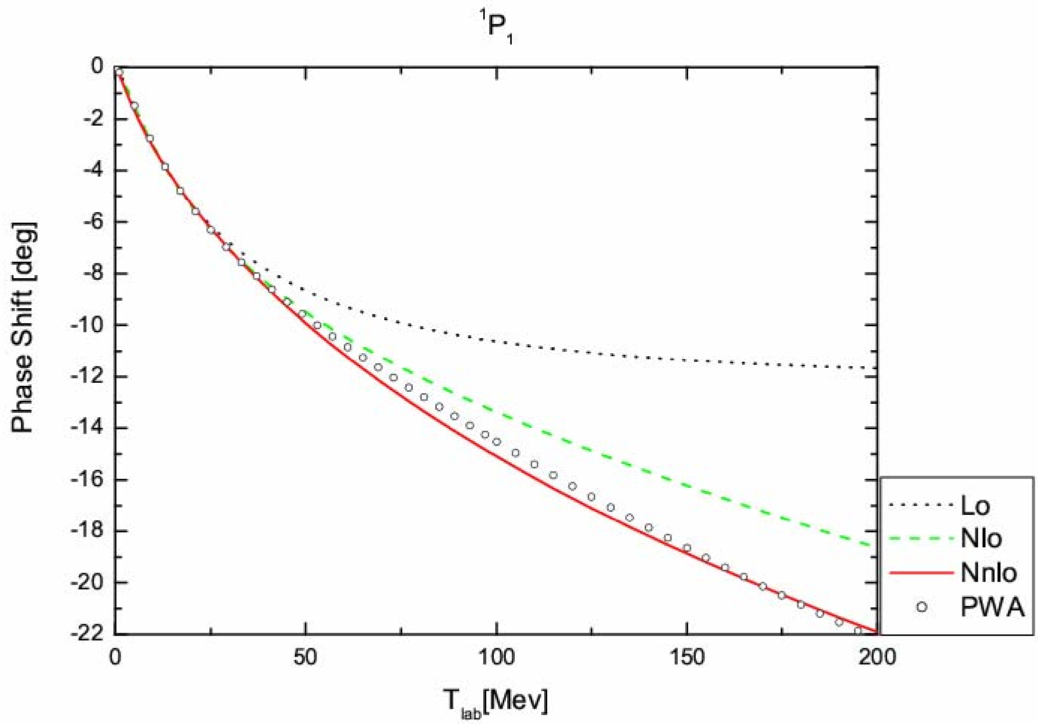
How to treat the nonperturbative renormalization prescriptions in such cases as rigorous solutions of T matrix are hard to reach? Many different approaches have been proposed and studied. To date, there is no complete consensus concerning the consistent treatment of power counting and renormalization. The most recent debates have been focused on how to work with the conventional EFT power counting in nonperturbative regimes and finally how to renormalize the NNEFT in nonperturbative regime in subtractive algorithm. There are even some mood of doubts on the applicability of the EFT approach to NN interactions and nuclear forces.

In this section, we introduce a nonperturbative approximation method to parameterize the nonperturbative prescription dependence of the T matrix. This is simply done by turning the LSE into the following parametrization of on-shell T: $1/T = 1/V - \tilde{G}$, $\tilde{G} = VG_0T/(VT)$. Here the nonperturbative factor $\tilde{G}$ exclusively assumes the information about nonperturbative renormalization due to the convolution with Green function G_0. As power like divergences dominate the convolution, we tentatively apply Padé approximant to the nonperturbative factor $\tilde{G}$ (arXiv: nucl-th/0310048, 0407090): $\tilde{G} \approx -i\frac{Mp}{4\pi} + \frac{\nu_0 + \nu_1 p^2 + \cdots}{\delta_0 + \delta_1 p^2 + \cdots}$. Obviously, (ν_i, δ_j) contain the renormalization prescription dependence exclusively besides EFT couplings. Thus (ν_i, δ_j) provide an approximate parametrization of the nonperturbative renormalization prescription. The strong prescription dependence is obvious: given the potential already constructed in EFT, different values of these numbers would definitely yield different T-matrix: only one set of these numbers are acceptable, other values would be physically incorrect.

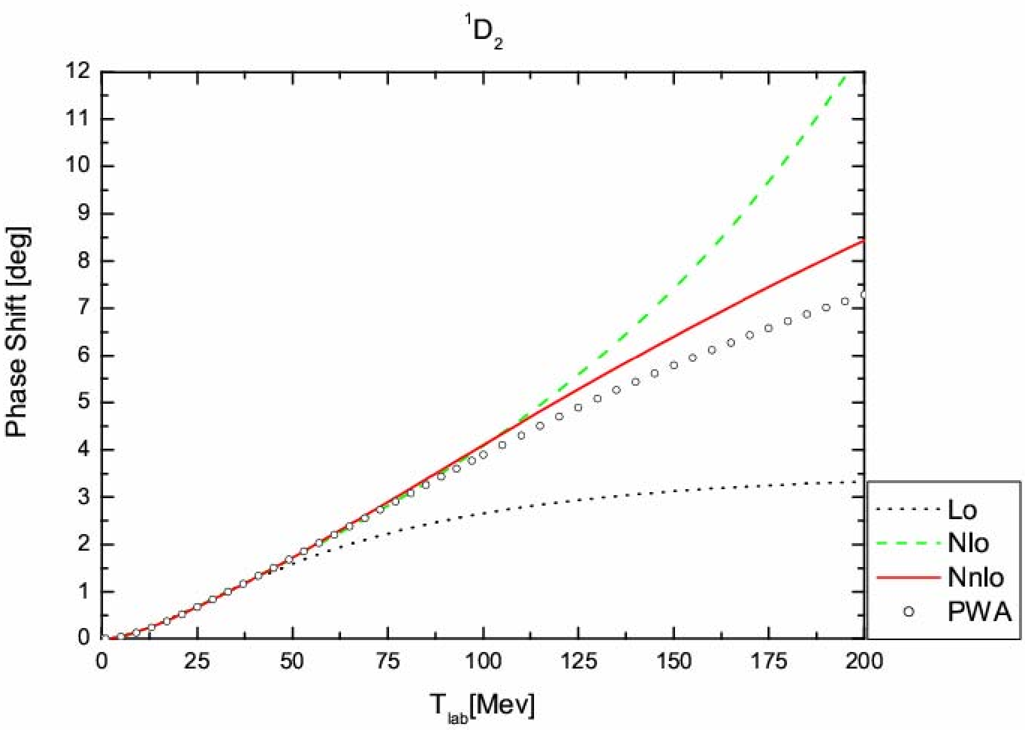
We have investigated the utility of such approximation approach in a number of channels to test the EFT applicability. Below are a small part of it, where the most coarse approximation of $\tilde{G}$, a constant g_0, is employed just to demonstrate our main points. We fit the low energy ends of the phase shift curves to PWA data in order to determine the Padé constants (ν_i, δ_j). The rationale for EFT approach are reflected in the following aspects: (a) Given a certain type of Padé approximant, higher order NN potentials would yield better phase shift curves in a general sense; (b) Given a certain order of potential, more sophisticated Padé approximant yields better theoretical predictions.

Phase shifts for 1S0, 1P1 and 1D2. For 1S0 channel, the empty circles are PWA data, while the phase shift curves computed with LO, NLO and NNLO potential are depicted with dotted, dashed and solid lines. The figure caption is clear for the rest two.





$^1P_1$
Phase Shift [deg]
T_{lab}[Mev]
Lo
Nlo
Nnlo
PWA



$^1D_2$
Phase Shift [deg]
T_{lab}[Mev]
Lo
Nlo
Nnlo
PWA

In the meantime, we could examine the order of magnitude of the Pade constants to test the rationality of the EFT approach to NN systems in a coarse manner. If the EFT is applicable the constant scale extracted from the $\tilde{G}$ factor should not fall outside of the EFT scope: $|g_0| < 1000000 Mev^2$. That is the scale $\sqrt{|g_0|} < 1000\ Mev$. In all the channels considered so far, this is true, in fact we have $\sqrt{|g_0|} \leq 600 Mev$. Of course, such a coarse approximation is far from being satisfactory, we could only find some demonstrative utility in it. More sophisticated Padé approximants are needed for more practical use.

Summary

1) Rigorous solutions of LSE within pionless EFT
2) Novel features and notions of renormalization in nonperturbative regime beyond conventional wisdoms
3) A simple nonperturbative scenario and the consequent predictions
4) A nonperturbative approximation parametrization of prescription

THANK YOU!