Radiative decay of $\Lambda_b \to \Lambda + \gamma$ in the standard model

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Outline

1. Motivation and introduction
2. Calculations of $\Lambda_b \to \Lambda + \gamma$ in PQCD
3. Numerical analysis of $\Lambda_b \to \Lambda + \gamma$ in PQCD
4. LCSR for the transition form factors of $\Lambda_b \to \Lambda + \gamma$
5. Numerical calculations of $\Lambda_b \to \Lambda + \gamma$ in LCSR
6. Discussions and conclusions

I. Motivation and introduction

- In the standard model (SM), flavor-changing neutral current (FCNC) processes are forbidden at the tree level and are strongly suppressed by the GIM mechanism, in particular for up-type quarks. Hence, they offer a unique possibility to test the CKM sector of the SM and possibly open a door to physics beyond the SM.

- In order to analyze the helicity structure of the effective Hamiltonian for the $b \to s$ transition in the SM and beyond the SM, investigation of the mesonic decays alone is not enough, since the information about the handedness of the quark is lost during the hadronization process.

- In order to maintain the helicity of the quarks, investigation of the baryonic decays is the only choice. For this reason, study of the baryonic decays receive special interest.

- From the viewpoint of experiment, the only drawback of bottom baryon decays is that the production rate of Λ_b baryon in b quark hadronization is about four times less than that of the B meson.

- There are some studies in the literature on $\Lambda_b \to \Lambda\gamma$ decays ranging from phenomenological models * to QCD sum rule approaches †. The existing theoretical predictions on $BR(\Lambda_b \to \Lambda\gamma)$ vary from each other and can be different even by orders of magnitude.

- In the first work, we apply the PQCD approach to study the radiative decay of $\Lambda_b \to \Lambda\gamma$, which can also serve as a new test for the applicability of k_T factorization theorem in baryon decays.

- In the second work, light-cone QCD sum rules approach is employed to investigate the decay of $\Lambda_b \to \Lambda\gamma$, in particular, the effect of higher twist LADAs of light Λ baryon to the transition form factor.

*T. Mannel and S. Recksiegel, J. Phys. **G24** (1998) 979.
R. Mohanta, A.K. Giri, M.P. Khanna, M. Ishida and S. Ishida, Prog. Theor. Phys. **102** (1999) 645.
H.Y. Cheng, C-Y. Cheung, G-L. Lin, Y.C. Lin, T.-M. Yan and H-L. Yu, Phys. Rev. **D51** (1995) 1199.

†Chao-Shang Huang and Hua-Gang Yan, Phys. Rev. **D59** (1999) 114022.

2. Calculations of $\Lambda_b \to \Lambda + \gamma$ in PQCD

- A short introduction to PQCD

- Effective Hamiltonian for $b \to s\gamma$

- Distribution amplitudes of Λ_b and Λ

- Factorization formulae of $\Lambda_b \to \Lambda + \gamma$

A short introduction to PQCD

- Factorization formulae in PQCD approach[‡]:

Taking the decay of $B \to M_1 M_2$ as an example, the amplitude can be factorized into the convolution of the six-quark hard kernel, the Wilson coefficient, the jet function and the Sudakov factor with the bound-state wave functions.

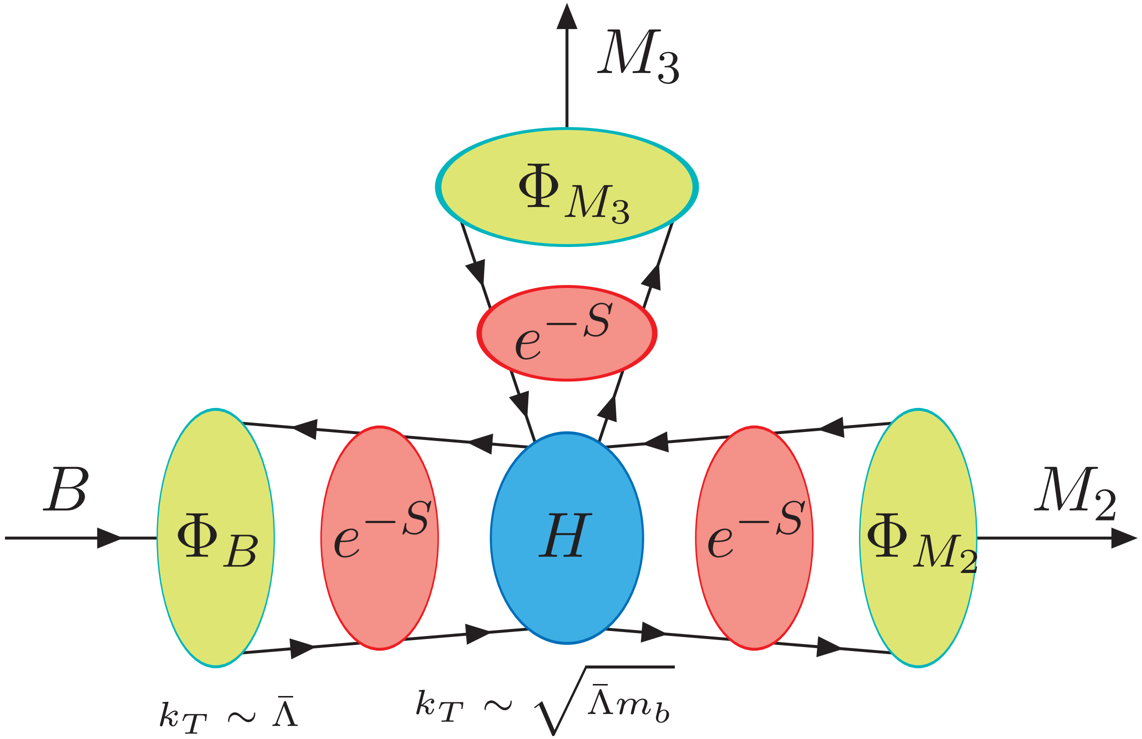
$$A = \phi_B \otimes H^{(6)} \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2} \, ,$$

all of which are well-defined and gauge-invariant.

J denotes the jet function from threshold resummation , which organizes the double logarithms $ln^2 x$ due to the radiative corrections to the hard kernel. S denotes the Sudakov factor from k_T resummation, which organizes the double logarithms $ln^2 k_T$ due to the radiative corrections to the meson wave function.

[‡]H. n. Li, Prog. Part. Nucl. Phys. **51** (2003) 85.

Graphics representation of PQCD factorization theorem:



Effective Hamiltonian for $b \to s\gamma$

- For $b \to s\gamma$, the effective Hamiltonian responsible for it can be written as:

$$H_{eff}(b \to s\gamma) \;=\; i\frac{G_F}{2\sqrt{2}}V_{tb}V_{ts}^{*}\frac{e}{4\pi^2}C_7^{eff}(\mu)[m_b\bar{s}\sigma_{\mu\nu}Rb + m_s\bar{s}\sigma_{\mu\nu}Lb]F^{\mu\nu}.$$

- The long-distance contributions to the decay of $\Lambda_b \to \Lambda\gamma$ are expected to be very small and can be neglected safely.

- The matrix element of magnetic penguin operators can be parameterized as

$$\begin{aligned}
\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}q^{\nu}b|\Lambda_b(P+q)\rangle &= f_2(0)\overline{\Lambda}(P)i\sigma_{\mu\nu}q^{\nu}\Lambda_b(P+q), \\
\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^{\nu}b|\Lambda_b(P+q)\rangle &= F_2(0)\overline{\Lambda}(P)i\sigma_{\mu\nu}\gamma_5 q^{\nu}\Lambda_b(P+q).
\end{aligned}$$

Hadronic wavefunctions

- Distribution amplitude of Λ_b:

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(k_i,\nu)$$
$$= \frac{1}{2\sqrt{2}N_c}\int \prod_{l=2}^{3} \frac{dw_l^+ d\mathbf{w_l}}{(2\pi)^3} e^{ik_l w_l} \varepsilon^{abc} \langle 0|T[b_\alpha^a(0) u_\beta^b(w_2) d_\gamma^c(w_3)]|\Lambda_b(p)\rangle$$
$$= \frac{f_{\Lambda_b}}{8\sqrt{2}N_c}[(\not p + M_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b(p)]_\alpha \Psi(k_i,\nu).$$

- Distribution amplitudes of Λ:

$$(Y_{\Lambda})_{\alpha\beta\gamma}(k_i',\nu)$$
$$= \frac{1}{2\sqrt{2}N_c}\int \prod_{l=1}^{2} \frac{dw_l^- d\mathbf{w_l}}{(2\pi)^3} e^{ik_l' w_l} \varepsilon^{abc} \langle 0|T[s_\alpha^a(w_1) u_\beta^b(w_2) d_\gamma^c(0)]|\Lambda(p')\rangle$$
$$= \frac{f_{\Lambda}}{8\sqrt{2}N_c}\{(\not p' C)_{\beta\gamma}[\gamma_5 \Lambda(p')]_\alpha \Phi^V(k_i',\nu) + (\not p' \gamma_5 C)_{\beta\gamma}[\Lambda(p')]_\alpha \Phi^A(k_i',\nu)\}$$
$$- \frac{f_{\Lambda}^T}{8\sqrt{2}N_c}(\sigma_{\mu\nu} p'^\nu C)_{\beta\gamma}[\gamma^\mu \gamma_5 \Lambda(p')]_\alpha \Phi^T(k_i',\nu).$$

- Evolution of wavefunctions and hard kernel:

$$\Psi(x_i, b_i, p, \nu) = \exp[-\sum_{l=2}^{3} s(\omega, x_l p^-) - 3\int_{\omega}^{\nu} \frac{d\overline{\mu}}{\overline{\mu}} \gamma_q(\alpha_s(\overline{\mu}))] \Psi(x_i),$$

$$\Phi^j(x_i', b_i', p', \nu) = \exp[-\sum_{l=1}^{3} s(\omega', x_l' p^+) - 3\int_{\omega'}^{\nu} \frac{d\overline{\mu}}{\overline{\mu}} \gamma_q(\alpha_s(\overline{\mu}))] \Phi^j(x_i'),$$

$$\omega = min(1/\tilde{b}_1, 1/\tilde{b}_2, 1/\tilde{b}_3),$$

$$\omega' = min(1/\tilde{b}_1', 1/\tilde{b}_2', 1/\tilde{b}_3').$$

$$H_{l,\mu}^{\alpha'\beta'\gamma'\alpha\beta\gamma}(x, x', b, b', M_{\Lambda_b}, \nu)$$
$$= \exp[-6\int_{\nu}^{t} \frac{d\overline{\mu}}{\overline{\mu}} \gamma_q(\alpha_s(\overline{\mu}))] \times H_{l,\mu}^{\alpha'\beta'\gamma'\alpha\beta\gamma}(x, x', b, b', M_{\Lambda_b}).$$

Factorization formulae of the decay amplitude

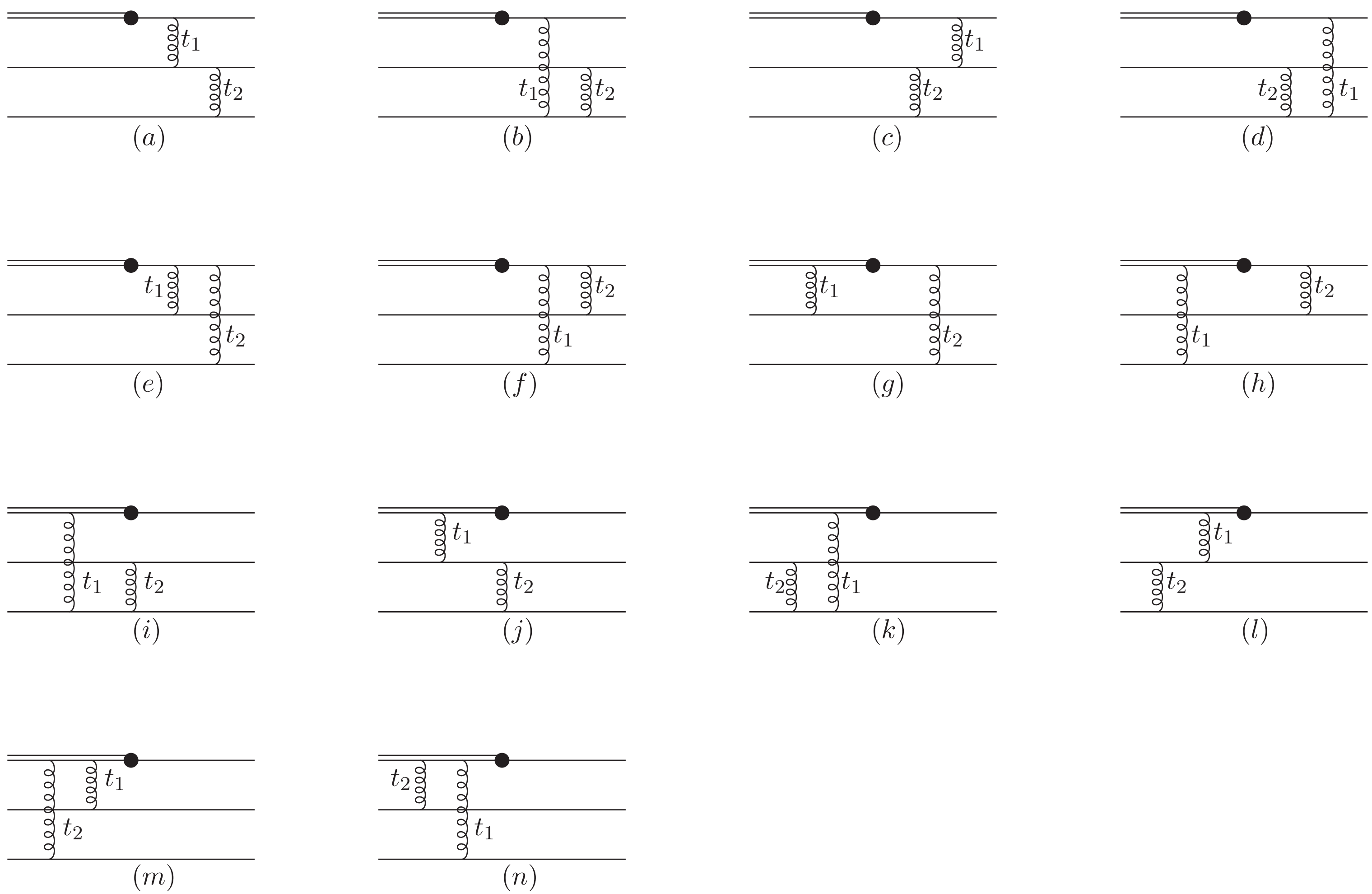
- The hadronic matrix elements can be written as:

$$M_{l,\mu} = \int [Dx] \int [Db] (\overline{Y}_{\Lambda})_{\alpha'\beta'\gamma'}(x'_i, b'_i, p', \nu)$$
$$H_{l,\mu}^{\alpha'\beta'\gamma'\alpha\beta\gamma}(x_i, x'_i, b_i, b'_i, M_{\Lambda_b}, \nu)(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i, b_i, p, \nu),$$

with

$$[Dx] = [dx][dx'], \quad [dx] = dx_1 dx_2 dx_3 \delta(1 - \sum_{l=1}^{3} x_l),$$
$$[dx'] = dx'_1 dx'_2 dx'_3 \delta(1 - \sum_{l=1}^{3} x'_l).$$

Feynman diagrams in the leading order of PQCD



3. Numerical analysis of $\Lambda_b \to \Lambda + \gamma$ in PQCD

- Distribution amplitudes of Λ_b and Λ

- Numerical results of $BR(\Lambda_b \to \Lambda + \gamma)$

- Discussions

Distribution amplitudes of Λ_b and Λ

- The Λ_b baryon distribution amplitude Ψ(Ref: F. Schlumpf, hep-ph/9211255):

$$\Psi(x_1, x_2, x_3) = N x_1 x_2 x_3 \exp[-\frac{M_{\Lambda_b}^2}{2\beta^2 x_1} - \frac{m_q^2}{2\beta^2 x_2} - \frac{m_q^2}{2\beta^2 x_3}].$$

Normalization condition:

$$\int [dx] \Psi(x_1, x_2, x_3) = 1.$$

β and m_q vary within ranges as $(0.6 \sim 1)$GeV and $(0.2 \sim 0.3)$GeV.

- The Λ baryon distribution amplitudes(Ref: COZ, Z. Phys. **C 42**, 569 (1989).):

$$\begin{aligned}
\phi^V(x_1, x_2, x_3) &= 42\phi_{as}(x_1, x_2, x_3)[0.18(x_3^2 - x_2^2) + 0.10(x_2 - x_3)], \\
\phi^A(x_1, x_2, x_3) &= -42\phi_{as}(x_1, x_2, x_3)[0.26(x_3^2 + x_2^2) \\
&\quad +0.34x_1^2 - 0.56x_2x_3 - 0.24x_1(x_2 + x_3)], \\
\phi^T(x_1, x_2, x_3) &= 42\phi_{as}(x_1, x_2, x_3)[1.2(x_2^2 - x_3^2) - 1.4(x_2 - x_3)], \\
\phi_{as}(x_1, x_2, x_3) &= 120x_1x_2x_3.
\end{aligned}$$

- The normalization constant f_{Λ_b} is determined by fitting $B(\Lambda_b \to \Lambda_c l \overline{\nu})$ whose central value is 5% measured by DELPHI detector at LEP using the same PQCD method.

f_{Λ_b}(GeV)	$\beta = 0.6$GeV	$\beta = 0.7$GeV	$\beta = 0.8$GeV	$\beta = 0.9$GeV	$\beta = 1$GeV
$m_q = 0.2$GeV	0.691×10^{-3}	0.841×10^{-3}	1.02×10^{-3}	1.21×10^{-3}	1.43×10^{-3}
$m_q = 0.3$GeV	1.27×10^{-3}	1.45×10^{-3}	1.65×10^{-3}	1.88×10^{-3}	2.12×10^{-3}

- The constants f_Λ and f_Λ^T are estimated by QCDSR as

$$f_\Lambda = 0.63 \times 10^{-2}\mathrm{GeV}^2, \quad f_\Lambda^T = 0.063 \times 10^{-2}\mathrm{GeV}^2.$$

Numerical results of $BR(\Lambda_b \to \Lambda\gamma)$

- $BR(\Lambda_b \to \Lambda\gamma)$ for different choices of β and m_q.

BR($\times 10^8$)	$\beta = 0.6$GeV	$\beta = 0.7$GeV	$\beta = 0.8$GeV	$\beta = 0.9$GeV	$\beta = 1$GeV
$m_q = 0.2$GeV	8.60	7.22	5.91	4.92	4.60
$m_q = 0.3$GeV	5.96	5.73	5.70	4.67	4.30

Correspondingly, $f_2(0) = F_2(0) = (1.2 \sim 1.6) \times 10^{-2}$ in PQCD.

- $BR(\Lambda_b \to \Lambda\gamma)$ calculated in other theoretical approaches:

Model	PM	COQM	HQET	BM	NRQM	QCDSR	PQCD	Exp.
BR ($\times 10^{-5}$)	$1.0 \sim 4.5$	0.23	$0.8 \sim 1.5$	0.4	0.27	3.7 ± 0.5	$(0.43 \sim 0.86) \times 10^{-2}$	≤ 130

- Only the upper bound 1.3×10^{-3} for $BR(\Lambda_b \to \Lambda\gamma)$ decay is available in experiment at present, so we have to wait for more experimental data to discriminate existing models.

References:

1. T. Mannel and S. Recksiegel, J. Phys. G **24** (1998) 979.

2. R. Mohanta, A. K. Giri, M. P. Khanna, M. Ishida and S. Ishida, Prog. Theor. Phys. **102** (1999) 645.

3.H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, T. M. Yan and H. L. Yu, Phys. Rev. D **51** (1995) 1199.

4. H. Y. Cheng and B. Tseng, Phys. Rev. D **53**, 1457 (1996) [Erratum-ibid. D **55**, 1697 (1997)].

5. C. S. Huang and H. G. Yan, Phys. Rev. D **59** (1999) 114022 [Erratum-ibid. D **61** (2000) 039901]

Discussions

- PQCD calculation for the branching ratio of $B \to K^* \gamma$ obtains a value of order consistent to other model calculations and also agrees with experimental value of about 4×10^{-5}.

- The branching ratio for $\Lambda_b \to \Lambda \gamma$ is expected to be smaller than that of $B \to K^* \gamma$ due to several suppression factors such as an additional α_s^2 and Sudakov suppression factor due to an additional spectator quark involved in the process.

- In the PQCD approach, both gluons are hard ones which excludes the possibility of including contributions where two spectator quarks are soft. The soft contributions to the form factors will be investigated in the light-cone sum rules approach.

4. Light-cone sum rules for the transition form factors of $\Lambda_b \to \Lambda + \gamma$

- A short introduction to LCSR

- Distribution amplitudes of light Λ baryon

- LCSR for the form factors of $\Lambda_b \to \Lambda + \gamma$

A short introduction to LCSR

- Light-cone sum rules (LCSR) were developed in late 80-th of last century in an attempt to solve or at least moderate the problems of three-point sum rules by making a partial resummation of the OPE to all orders and reorganizing the expansion in terms of twist of relevant operators rather than their dimension.

- The difference between LCSR and SVZSR is that the expansion at short distances is substituted by the expansion in the transverse distance between partons in the infinite momentum frame.

- Technically, the LCSR approach presents a marriage of QCD sum rules with the theory of hard exclusive processes.

- One advantage of LCSR approach is that the soft non-perturbative contribution to the transition form factor can be calculated quantitatively.

Distribution amplitudes of light Λ baryon

- The distribution amplitudes of Λ barton up to leading Fock state can be defined by the non-local matrix element, similar to the case for the nucleon, as §

$$
\begin{aligned}
&4\langle 0|\epsilon^{ijk}u_\alpha^i(a_1 z)d_\beta^j(a_2 z)s_\gamma^k(a_3 z)|\Lambda(P)\rangle \\
&= \mathcal{A}_1\,(\not{P}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_2 M\,(\not{P}\gamma_5 C)_{\alpha\beta}\,(\not{z}\Lambda)_\gamma + \mathcal{A}_3 M\,(\gamma_\mu\gamma_5 C)_{\alpha\beta}\,(\gamma^\mu\Lambda)_\gamma \\
&+\mathcal{A}_4 M^2\,(\not{z}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_5 M^2\,(\gamma_\mu\gamma_5 C)_{\alpha\beta}\,(i\sigma^{\mu\nu}z_\nu\Lambda)_\gamma + \mathcal{A}_6 M^3\,(\not{z}\gamma_5 C)_{\alpha\beta}\,(\not{z}\Lambda)_\gamma\,.
\end{aligned}
$$

- The invariant functions $\mathcal{S}_i$, $\mathcal{P}_i$,$\mathcal{V}_i$, $\mathcal{A}_i$,$\mathcal{T}_i$ can be related to the distribution amplitudes S_i, P_i, V_i, A_i and T_i,as

$$
\begin{aligned}
\mathcal{A}_1 &= A_1\,, & 2p\cdot z\mathcal{A}_2 &= -A_1 + A_2 - A_3\,,\\
2\mathcal{A}_3 &= A_3\,, & 4p\cdot z\mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5\,,\\
4p\cdot z\mathcal{A}_5 &= A_3 - A_4\,, & (2p\cdot z)^2\mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6\,.
\end{aligned}
$$

§V. Braun, R.J. Fries, N. Mahnke and E. Stein, Nucl. Phys. **B589** (2000) 381 [Erratum-ibid. B **607** (2001) 433] [arXiv:hep-ph/0007279].
Min-Qiu Huang and Dao-Wei Wang, arXiv: hep-ph/0608170.

- Up to leading conformal spin accuracy, the light-cone distribution amplitudes A_i can be expressed

$$
\begin{aligned}
A_1(x_1,x_2,x_3) &= -120x_1x_2x_3\phi_3^0,\\
A_2(x_1,x_2,x_3) &= -24x_1x_2\phi_4^0,\\
A_3(x_1,x_2,x_3) &= -12x_3(1-x_3)\psi_4^0,\\
A_4(x_1,x_2,x_3) &= -3(1-x_3)\phi_5^0,\\
A_5(x_1,x_2,x_3) &= -6x_3\phi_5^0,\\
A_6(x_1,x_2,x_3) &= -2\phi_6^0.
\end{aligned}
$$

- At the case of Λ baryon, all the 6 parameters can be expressed in terms of 2 independent matrix elements of local operators:

$$\phi_3^0=\phi_6^0=-f_\Lambda,\ \phi_4^0=\phi_5^0=-\frac{1}{2}(\lambda_1+f_\Lambda)\,,\ \psi_4^0=\psi_5^0=-\frac{1}{2}(\lambda_1-f_\Lambda)\,,$$

with

$$f_\Lambda=6.1\times10^{-3}\mathrm{GeV}^2,\ \ \lambda_1=-1.2\times10^{-2}\mathrm{GeV}^2.$$

LCSR for the transition form factors of $\Lambda_b \to \Lambda + \gamma$

- The correction function for $\Lambda_b \to \Lambda + \gamma$ can be chosen as

$$z^\nu T_\nu(P,q) = i z^\nu \int d^4x e^{-iq\cdot x} \langle 0|T\{j_{\Lambda_b}(0) j_\nu(x)\}|\Lambda(P)\rangle,$$

where the current $j_{\Lambda_b}(0)$ and $j_\nu(x)$ are given by

$$\begin{aligned} j_{\Lambda_b}(0) &= \epsilon^{ijk}[u^i(0) C\gamma_5 \not{z} d^j(0)] \not{z} b^k(0) , \\ j_\nu(x) &= i\bar{b}(x)\sigma_{\mu\nu}(1-\gamma_5) q^\mu s(x) . \end{aligned}$$

- The coupling of the chosen current with Λ_b can be given by

$$\langle 0|j_{\Lambda_b}(0)|\Lambda_b(P')\rangle = f_{\Lambda_b}(z\cdot P') \not{z}\Lambda_b(P').$$

- The correction function can be calculated in the hadron level as

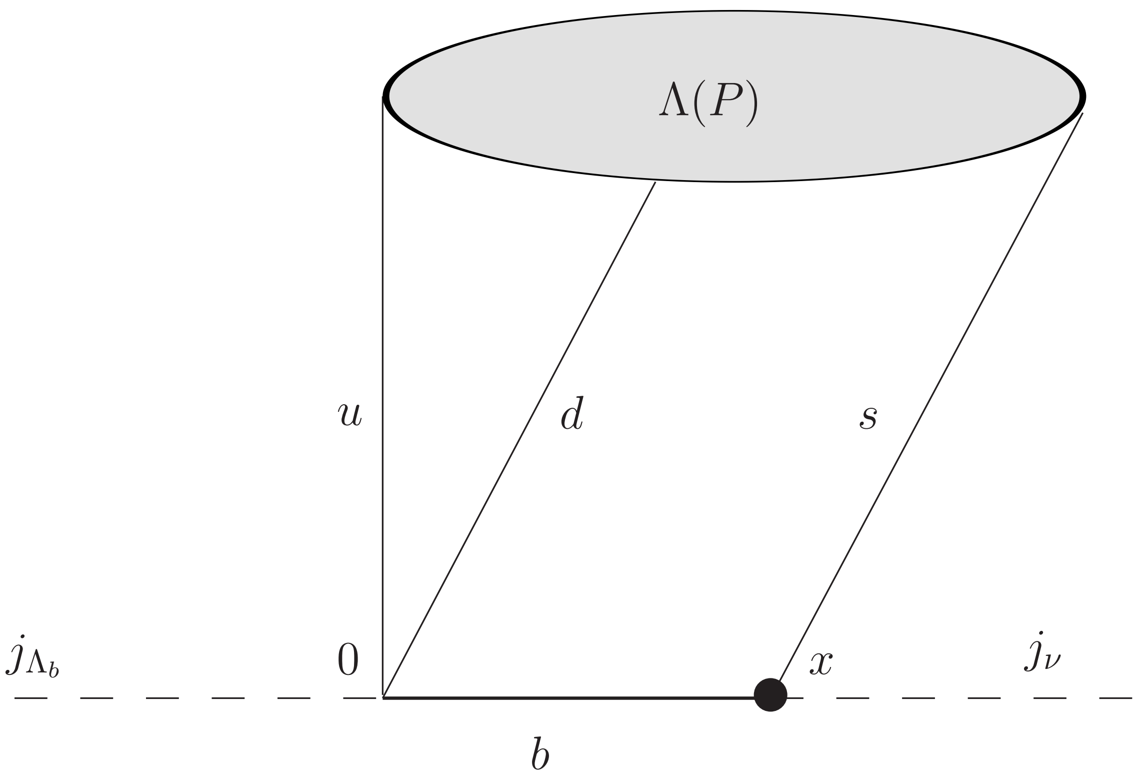
$$z^\nu T_\nu = 2f_{\Lambda_b}\frac{(z\cdot P')^2}{m_{\Lambda_b}^2 - P'^2}[-f_1 \not{z} + f_2 \not{z} \not{q} - F_1 \not{z}\gamma_5 - F_2 \not{z} \not{q}\gamma_5]\Lambda(P) + ...,$$

where $P' = P + q$ and the ellipses denote the terms in proportion to the higher power of $1/P$ in the infinite momentum kinematics $P \sim \infty$, $q \sim const$, $z \sim 1/P$ and the contributions from the higher states of Λ_b channel.

- To the leading order of α_s and making use of the OPE technique, the correlation function can be computed as

$$\begin{aligned} z^\nu T_\nu \ &= \ -2(C\gamma_5 \not{z})_{\alpha\beta}[\not{z}(1-\gamma_5)]_\gamma \\ &\int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(k-q)\cdot x}\frac{z\cdot k}{k^2 - m_b^2}\langle 0|\epsilon^{ijk}u_\alpha^i(0)d_\beta^j(0)s_\gamma^k(x)|\Lambda(P)\rangle, \end{aligned}$$

which can be represented by the following figure intuitionally.


$\Lambda(P)$
u
d
s
j_{Λ_b}
0
x
j_ν
b

- Matching the correlation function obtained in the hadron level and quark representation, we can derive the sum rules for the form factors as

$$
\begin{aligned}
f_{\Lambda_b} f_2(q^2) e^{-m_{\Lambda_b}^2/M_B^2} &= \int_{x_0}^{1} dx_3 e^{-s/M_B^2} \Big[\Big(\int_0^{1-x_3} dx_3 A_1(x_1, 1-x_1-x_3, x_3) \Big) \\
&\quad - \frac{M^2}{M_B^2} \Big(-2\tilde{A}_1(x_3) + \tilde{A}_2(x_3) - \tilde{A}_3(x_3) - \tilde{A}_4(x_3) + \tilde{A}_5(x_3) \Big) \\
&\quad + \frac{M^4}{M_B^4} \Big(\tilde{\tilde{A}}_1(x_3) - \tilde{\tilde{A}}_2(x_3) + \tilde{\tilde{A}}_3(x_3) + \tilde{\tilde{A}}_4(x_3) - \tilde{\tilde{A}}_5(x_3) + \tilde{\tilde{A}}_6(x_3) \Big) \Big] \\
&\quad - \frac{M^2 x_0^2 e^{-s_0/M_B^2}}{m_b^2 - q^2 + x_0^2 M^2} \Big[\Big(-2\tilde{A}_1(x_0) + \tilde{A}_2(x_0) - \tilde{A}_3(x_0) - \tilde{A}_4(x_0) + \tilde{A}_5(x_0) \Big) \\
&\quad - \frac{M^2}{M_B^2} \Big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0) \Big) \\
&\quad + M^2 \frac{d}{dx_0} \Big(\frac{x_0^2 \big(\tilde{\tilde{A}}_1(x_0) - \tilde{\tilde{A}}_2(x_0) + \tilde{\tilde{A}}_3(x_0) + \tilde{\tilde{A}}_4(x_0) - \tilde{\tilde{A}}_5(x_0) + \tilde{\tilde{A}}_6(x_0) \big)}{m_0^2 - q^2 + x_0^2 M^2} \Big) \Big], \\
F_2(q^2) &= f_2(q^2),
\end{aligned}
$$

where

$$
s = (1-x_3) M^2 + \frac{m_b^2 + (1-x_3) Q^2}{x_3},
$$

and

$$
x_0 = \frac{\sqrt{(Q^2 + s_0 - M^2)^2 + 4M^2(Q^2 + m_b^2)} - (Q^2 + s_0 - M^2)}{2M^2}.
$$

5. Numerical calculations of $\Lambda_b \to \Lambda + \gamma$ in LCSR

- Input parameters, Borel mass and threshold value.

- Transition form factors for $\Lambda_b \to \Lambda + \gamma$

- Decay width and Λ polarization asymmetry of $\Lambda_b \to \Lambda + \gamma$

Input parameters

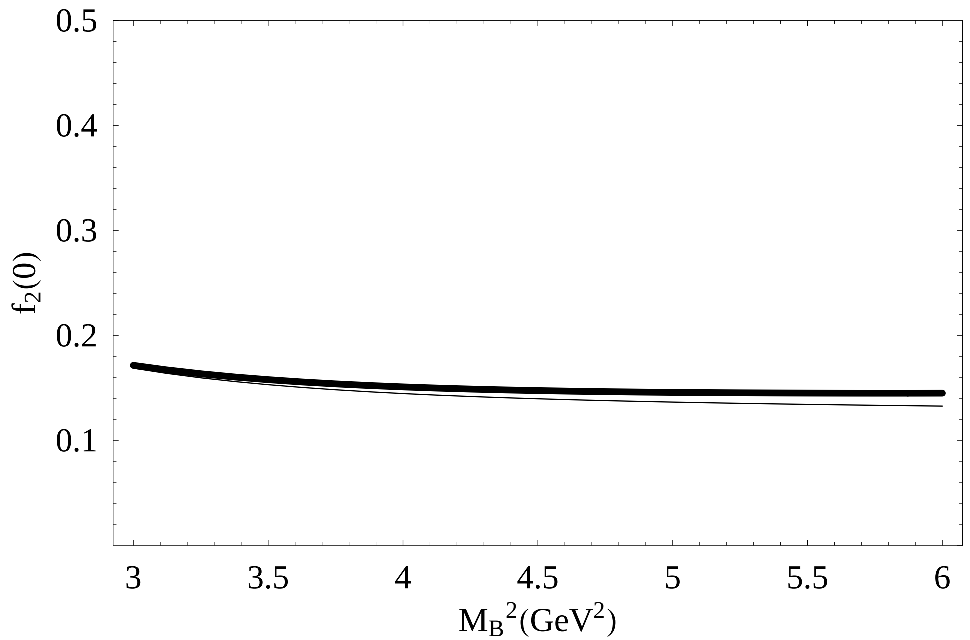
- The input parameters used in this work can be grouped as below

$$
\begin{array}{ll}
m_b = (4.68 \pm 0.03)\mathrm{GeV}, & m_s(1\mathrm{GeV}) = 142\mathrm{MeV}, \\
m_{\Lambda_b} = 5.62\mathrm{GeV}, & m_\Lambda = 1.12\mathrm{GeV}, \\
f_{\Lambda_b} = 3.9 \times 10^{-3}\mathrm{GeV}^2, & f_\Lambda = 6.1 \times 10^{-3}\mathrm{GeV}^2 \\
\lambda_1 = -1.2 \times 10^{-2}\mathrm{GeV}^2, & s^0_{\Lambda_b} = 39 \pm 1\mathrm{GeV}^2.
\end{array}
$$

- We require the contributions from the higher states to be less than 30 % and the value of the form factors $f_2(0)$ and $F_2(0)$ does not vary drastically within the selected region for the Borel masses. In this way, we indeed find a Borel platform $M_B^2 \in [3.0, 6.0]\mathrm{GeV}^2$ as showed in the below figure.

Transition form factor for $\Lambda_b \to \Lambda + \gamma$

- The numbers of $f_2(0)$ within the Borel window.



- Numerical results for the parameters for $\xi_i(0)$.

	twist-3	up to twist-6	COZ	FZOZ
$f_2(0)$	$0.14^{+0.02}_{-0.01}$	$0.15^{+0.02}_{-0.02}$	$0.74^{+0.06}_{-0.06}$	$0.87^{+0.07}_{-0.07}$

In the COZ model, the leading twist DAs A_1 can be written as

$$\begin{aligned} A_1^{COZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\times[0.26(x_3^2+x_2^2)+0.34x_1^2-0.56x_2x_3-0.24x_1(x_2+x_3)], \\ \phi_{as}(x_1,x_2,x_3) &= 120x_1x_2x_3. \end{aligned}$$

While in the FZOZ model, the manifest expression of A_1 can be written as

$$\begin{aligned} A_1^{FZOZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\times[0.093(x_3^2+x_2^2)+0.376x_1^2-0.194x_2x_3-0.207x_1(x_2+x_3)], \end{aligned}$$

- It seems that the form factors responsible for $\Lambda_b \to \Lambda$ transition may be dominated by the soft gluons exchange between valence quarks inside the Λ_b and Λ baryons.

Decay width of $\Lambda_b \to \Lambda + \gamma$ and polarization asymmetry of Λ baryon

- The decay width of $\Lambda_b \to \Lambda + \gamma$ can be derived as

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{\alpha_{em} G_F^2}{64 m_{\Lambda_b}^3 \pi^4} |V_{tb}|^2 |V_{ts}|^2 |C_7^{eff}|^2 (1-x^2)^3 (m_b^2 + m_s^2) [f_2(0)]^2 \times [1 + \frac{2x}{1-x^2} \frac{m_b^2 - m_s^2}{m_b^2 + m_s^2} (v \cdot s)],$$

where $x = m_\Lambda / m_{\Lambda_b}$ and $v = p_{\Lambda_b} / m_{\Lambda_b}$.

- The four-spin vector of Λ baryon in the rest frame of Λ_b can be written as

$$s_0 = \frac{\mathbf{p}_\Lambda \cdot \hat{\xi}}{m_\Lambda}, \qquad \mathbf{s} = \hat{\xi} + \frac{s_0}{E_\Lambda + m_\Lambda} \mathbf{p}_\Lambda,$$

where $\hat{\xi}$ is a unit vector along the spin direction of Λ in its rest frame.

$$v \cdot s = \frac{1-x^2}{1+x^2} \hat{\mathbf{p}} \cdot \mathbf{s} = \frac{1-x^2}{2x} \hat{\mathbf{p}} \cdot \hat{\xi},$$

where $\hat{\mathbf{p}}$ is a unite momentum vector of Λ baryon.

- For the convenience of comparing with the experiment, we rewrite the radiative decay rate of Λ_b into polarized Λ baryon as [¶]

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{1}{2}\Gamma_0[1 + \alpha\ \hat{\mathbf{p}} \cdot \mathbf{s}] = \frac{1}{2}\Gamma_0[1 + \alpha'\ \hat{\mathbf{p}} \cdot \hat{\xi}],$$

with

$$\alpha = \frac{2x}{1+x^2}\frac{m_b^2 - m_s^2}{m_b^2 + m_s^2} = 0.38, \qquad \alpha' = \frac{m_b^2 - m_s^2}{m_b^2 + m_s^2} = 1.$$

- The total decay width of $\Lambda_b \to \Lambda + \gamma$ in the light-cone sum rules.

Model of DAs	twist-3	up to twist-6	COZ	FZOZ
BR	$0.63^{+0.17}_{-0.12} \times 10^{-5}$	$0.73^{+0.15}_{-0.15} \times 10^{-5}$	$1.8^{+0.3}_{-0.3} \times 10^{-4}$	$2.6^{+0.4}_{-0.4} \times 10^{-4}$

[¶] 1. C. S. Huang and H. G. Yan, Phys. Rev. D **59** (1999) 114022 [Erratum-ibid. D **61** (2000) 039901].
2. C.K. Chua, X.G. He and W.S. Hou, Phys.Rev. D60 (1999) 014003.

6. Discussions and conclusions

- In this work, we explore the soft contributions to the form factors responsible for the $\Lambda_b \to \Lambda$ transition in terms of LCSR approach to understand the tremendous discrepancy on predictions of decay rate for $\Lambda_b \to \Lambda + \gamma$ between theoretical methods.

- We confirm that the Λ polarization asymmetry of $\Lambda_b \to \Lambda\gamma$ only relies on the relative strength of left- and right- handed couplings between quarks and is free of the pollution from the strong interaction.

- Theoretical predictions presented here can be systemically improved by including the higher conformal spin contributions as well as radiative corrections in the sum rules for the transition form factors.

- A systematic study of baryon distribution amplitudes is mandatory.

Thanks very much for your attentions!

BACKUP!

Nonperturbative methods for calculating the form factors

Lattice QCD:

Quark model:
The dynamics of the form factor is represented by the non-perturbative parameters in the hadronic wavefunction .

Perturbative QCD:
The form factor can be expressed as a convolution of the hadronic wavefunctions with hard scattering kernels.

(light-cone) QCD sum rules:
The form factor can be expressed by some universal parameters, such as condensates of quark and gluon, light-cone distribution amplitude.

Bethe- Salpeter equation, Potential model, Bag model, Skyrme model, ...

A short introduction to PQCD

- Wigner-Eckart theorem versus factorization in QCD:

  The Wigner-Eckart theorem states that the matrix element $\langle\alpha'; j', m'|Q_q^{(k)}|\alpha; j, m\rangle$ depends on the quantum numbers m, m' and q only through Clebsch-Gordan coefficients, specifically

  $$\langle\alpha'; j', m'|Q_q^{(k)}|\alpha; j, m\rangle = C(j', m'|j, k; m, q)\, \langle\alpha'; j'\|Q^k\|\alpha; j\rangle \ ,$$

  where $C(j', m'|j, k; m, q)$ is the corresponding Clebsch-Gordan coefficient and the object $\langle\alpha'; j'\|Q^k\|\alpha; j\rangle$ is also called *reduced matrix element*

  Factorization amounts to showing that the long-distance contributions to the matrix elements $\langle[c1][c2]|O_i|[s]\rangle$ are actually contained in the simpler matrix elements $\langle[c1]|(\bar{q}b)(0)|[s]\rangle$ (form factors), $\langle[ci]|\bar{q}(x)q(0)|0\rangle$, and $\langle 0|\bar{q}(x)b(0)|[s]\rangle$ (light-cone distribution amplitudes). It is then assumed that if this holds perturbatively to all orders for all quark-gluon matrix elements, then it does for the hadronic matrix elements [‖]

[‖] M. Beneke, eConf **C0610161** (2006) 030 [Nucl. Phys. Proc. Suppl. **170** (2007) 57] [arXiv:hep-ph/0612353].

- Collinear factorization and k_T factorization:

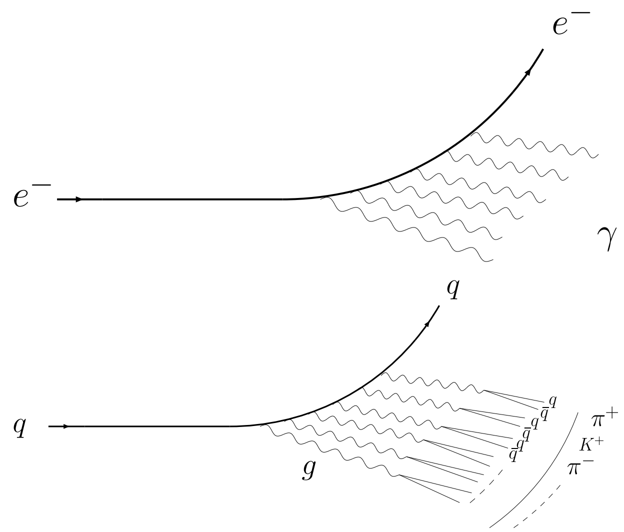
  Collinear factorization:
  QCD factorization (Beneke, Buchalla, Neubert, Sachajda, ...),
  Light-cone sum rules(Chernyak, Zhitnitsky, Balitsky, Braun, ...),
  Soft-collinear effective theory(Bauer, Pirjol, Rothstein, Stewart,..).

  k_T factorization:
  PQCD approach(Keum, Li, Sanda, Lü, Ukai, Yang, Xiao, ...).
  The transition form factors in PQCD approach are dominated by the hard gluon exchange corresponding to the hard rescattering mechanism. Soft contribution, though indeed playing a role, is less important because of suppression from the Sudakov mechanism. Unlike QCD sum rules, soft contribution can not be included into the PQCD formalism in a consistent way: if there is no hard gluon exchange to provide a large characteristic scale, twist expansion does not hold. Therefore, soft contribution can not be estimated using the same meson distribution amplitudes resulting from twist expansion.

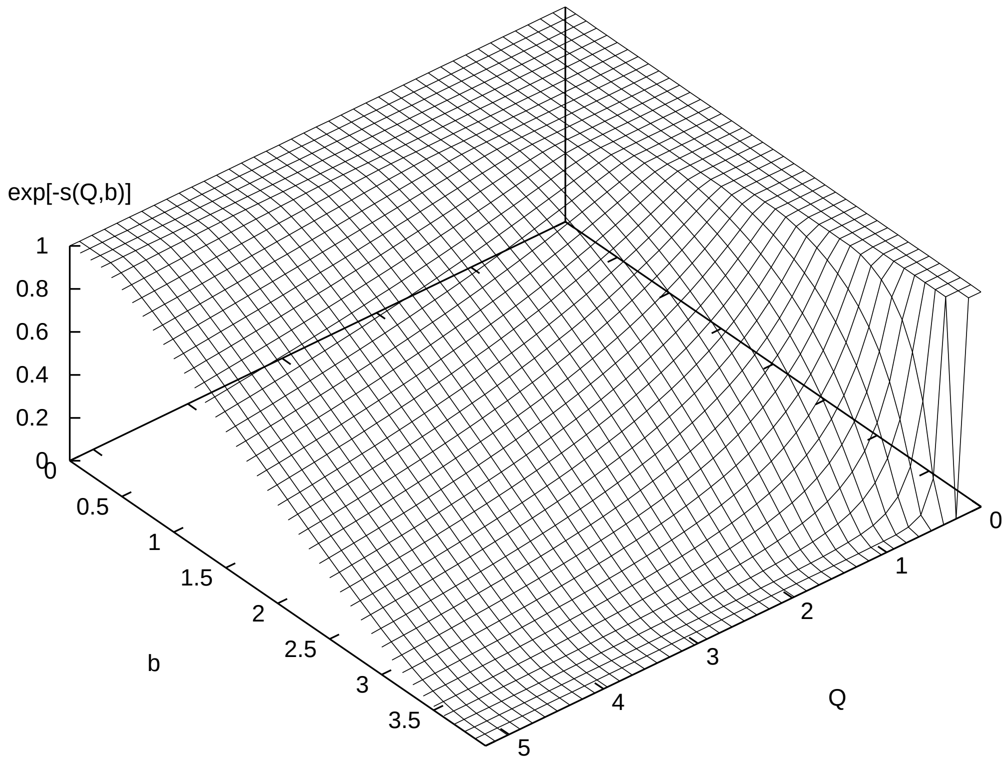
Physical interpretation of Sudakov factor

- In order to understand the Sudakov factor physically, first we consider QED. When a charged particle is accelerated, infinitely many photons must be emitted by the bremsstrahlung.

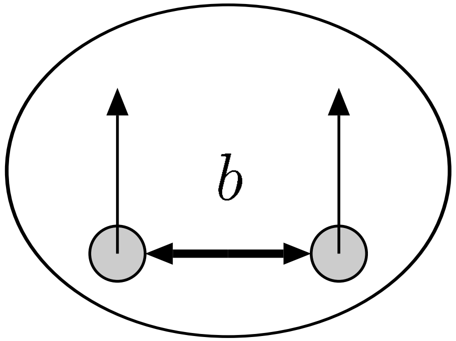
  A similar phenomenon occurs when a quark is accelerated: infinitely many gluons must be emitted. According to the feature of strong interaction, gluons cannot exist freely, so hadronic jet is produced. Then we observe many hadrons in the end if gluonic bremsstrahlung occurs.



- The amplitude for an exclusive decay $B \to M_1 M_2$ is proportional to the probability that no bremsstrahlung gluon is emitted. This is the Sudakov factor and it is depicted in the following figure.



- As can be seen, the Sudakov factor is large for small b and Q. Large b implies that the quark and antiquark pair is separated, which in turn implies less color shielding (see the figure below). Similar absence of shielding occurs when b quark carries most of the momentum while the momentum fraction of spectator quark x in the B meson is small.



Then the Sudakov factor suppresses the long distance contributions for the decay process and gives the effective cutoff about the transverse direction. In short, the Sudakov factor corresponds to the probability for emitting no photons. According to this factor, the property of short distance is guaranteed.

Truncation on Sudakov factor

As can be seen from the figure of Sudakov factor, there is no suppression for small b, where the two fermion lines are close to each other.

For the simplicity of analysis, we set the Sudakov factor exponential to unity in the small b region where it induce a (small) enhancement,

since in this region it is dominated by low orders in perturbative theory, and should be thought of as a part of the higher-order corrections to the hard kernel.

Manifest expressions of Sudakov factor

The explicit form for the function $s(Q,b)$ is:

$$s(Q,b) = \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}\left(\hat{q}-\hat{b}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)$$
$$+\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right],$$

where the variables are defined by

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \ln[1/(b\Lambda)],$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1 = \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24},$$
$$A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\ln(\frac{1}{2}e^{\gamma_E}),$$

n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop running coupling constant, i.e. we pick up the four terms in the first line of the expression for the function $s(Q,b)$.

Kinematics of $\Lambda_b \to \Lambda\gamma$

- We define, in the rest frame of Λ_b, p, p' to be the Λ_b, Λ momenta.
  $k_i (i = 1, 2, 3)$ to be the valence quark momenta inside Λ_b.
  k'_i to be the valence quark momenta inside Λ.

$$p = (p^+, p^-, \mathbf{0}_T) = \frac{M_{\Lambda_b}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad p' = (p'^+, 0, \mathbf{0}_T)$$

$$k_1 = (p^+, x_1 p^-, \mathbf{k}_{1T}), \quad k_2 = (0, x_2 p^-, \mathbf{k}_{2T}), \quad k_3 = (0, x_3 p^-, \mathbf{k}_{3T})$$

$$k'_1 = (x'_1 p'^+, 0, \mathbf{k}'_{1T}), \quad k'_2 = (x'_2 p'^+, 0, \mathbf{k}'_{2T}), \quad k'_3 = (x'_3 p'^+, 0, \mathbf{k}'_{3T}).$$

One can write $p'^+ = \rho p^+$ with $\rho = \frac{2p\cdot p'}{M^2_{\Lambda_b}} = \frac{p^2 + p'^2 - q^2}{M^2_{\Lambda_b}}$.

In the case of $\Lambda_b \to \Lambda\gamma$, $q^2 = 0$.

A short introduction to QCD sum rules

- The basic idea of QCD sum rules is calculating the correlation functions in the quark and hadron levels respectively, and then matching them with the assumption of quark-hadron duality (SVZ, 1979).

- The QCD representation of the correlation function is calculated in the framework of operator product expansion (OPE), where the short- and long-distance quark-gluon interactions are separated.
  The former can be computed using QCD perturbative theory, whereas the latter can be parameterized by a few vacuum expectation values of composite operators (Colangelo and Khodjamirian, hep-ph/0010175).

- The hadronic representation of the correlation function can be derived by inserting the complete set of states between the currents in the correlator.

- Applications of QCD sum rules:

  (a)determination of the light and heavy quark masses;

  (b) masses and decay constants of light and heavy mesons and baryons;

  (c) form factors of mesons and baryons;

  (d) moments of distribution amplitudes of light mesons and bayons;

  (e)strong couplings and magnetic moments of mesons and baryons;

  (f)spectroscopy and properties of exotic hadrons (gluball, hybrids...)

  ......

Problems of three-point SVZ sum rules

- One major problem of three-point SVZ sum rules is that OPE (short-distance expansion in condensates) upsets power counting in the large momentum or mass ** .

- Another major problem of three-point SVZ sum rules is the contamination of the sum rules, at zero momentum transfer, by "nondiagonal" transition of the ground state to excited states.

**V. M. Braun, arXiv:hep-ph/9801222.

Distribution amplitudes of light Λ baryon

- The distribution amplitudes of Λ barton up to leading Fock state can be defined by the non-local matrix element, similar to the case for the nucleon, as [††]

$$
\begin{aligned}
&4\langle 0|\epsilon^{ijk}u_\alpha^i(a_1 z)d_\beta^j(a_2 z)s_\gamma^k(a_3 z)|\Lambda(P)\rangle \\
&= \mathcal{S}_1 M C_{\alpha\beta}(\gamma_5\Lambda)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta}(\not{z}\gamma_5\Lambda)_\gamma + \mathcal{P}_1 M(\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{P}_2 M^2(\gamma_5 C)_{\alpha\beta}(\not{z}\Lambda)_\gamma \\
&+\mathcal{V}_1(\not{P}C)_{\alpha\beta}(\gamma_5\Lambda)_\gamma + \mathcal{V}_2 M(\not{P}C)_{\alpha\beta}(\not{z}\gamma_5\Lambda)_\gamma + \mathcal{V}_3 M(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu\gamma_5\Lambda)_\gamma \\
&+\mathcal{V}_4 M^2(\not{z}C)_{\alpha\beta}(\gamma_5\Lambda)_\gamma + \mathcal{V}_5 M^2(\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu}z_\nu\gamma_5\Lambda)_\gamma + \mathcal{V}_6 M^3(\not{z}C)_{\alpha\beta}(\not{z}\gamma_5\Lambda)_\gamma \\
&+\mathcal{A}_1(\not{P}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_2 M(\not{P}\gamma_5 C)_{\alpha\beta}(\not{z}\Lambda)_\gamma + \mathcal{A}_3 M(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\Lambda)_\gamma \\
&+\mathcal{A}_4 M^2(\not{z}\gamma_5 C)_{\alpha\beta}\Lambda_\gamma + \mathcal{A}_5 M^2(\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}z_\nu\Lambda)_\gamma + \mathcal{A}_6 M^3(\not{z}\gamma_5 C)_{\alpha\beta}(\not{z}\Lambda)_\gamma \\
&+\mathcal{T}_1(P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Lambda)_\gamma + \mathcal{T}_2 M(z^\mu P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma_5\Lambda)_\gamma + \mathcal{T}_3 M(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\gamma_5\Lambda)_\gamma \\
&+\mathcal{T}_4 M(P^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}z_\rho\gamma_5\Lambda)_\gamma + \mathcal{T}_5 M^2(z^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\gamma_5\Lambda)_\gamma \\
&+\mathcal{T}_6 M^2(z^\mu P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\not{z}\gamma_5\Lambda)_\gamma + \mathcal{T}_7 M^2(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\not{z}\gamma_5\Lambda)_\gamma \\
&+\mathcal{T}_8 M^3(z^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}z_\rho\gamma_5\Lambda)_\gamma\,.
\end{aligned}
$$

[††] V. Braun, R.J. Fries, N. Mahnke and E. Stein, Nucl. Phys. **B589** (2000) 381 [Erratum-ibid. B **607** (2001) 433] [arXiv:hep-ph/0007279].
Min-Qiu Huang and Dao-Wei Wang, arXiv: hep-ph/0608170.

It should be noted that the invariant functions defined in the above equation do not have a definite twist.

- The invariant functions $\mathcal{S}_i$, $\mathcal{P}_i$,$\mathcal{V}_i$, $\mathcal{A}_i$,$\mathcal{T}_i$ can be related to the distribution amplitudes S_i, P_i, V_i, A_i and T_i,as

$$
\begin{aligned}
\mathcal{A}_1 &= A_1\,, & 2p\cdot z\mathcal{A}_2 &= -A_1 + A_2 - A_3\,,\\
2\mathcal{A}_3 &= A_3\,, & 4p\cdot z\mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5\,,\\
4p\cdot z\mathcal{A}_5 &= A_3 - A_4\,, & (2p\cdot z)^2\mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6\,.
\end{aligned}
$$

Here only the relations associated with axial-vector distribution amplitudes are presented.

- The distribution amplitude $F_i(p\cdot z)$ can be wtitten as

$$F(a_i P\cdot z) = \int \mathcal{D}x e^{-ip\cdot z\sum_i x_i a_i} F(x_i)\ ,$$

where $\int \mathcal{D}x$ is defined by

$$\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)\,.$$

Definitions of f_Λ and λ_1

The coupling constant f_Λ and parameter λ_1 are defined by

$$\begin{aligned}\langle 0|\epsilon^{ijk}[u^i(0)C\gamma_5 \not{z} d^j(0)] \not{z} s^k(0)|\Lambda(P)\rangle &= f_\Lambda z \cdot P \not{z}\Lambda(P), \\ \langle 0|\epsilon^{ijk}[u^i(0)C\gamma_5\gamma_\mu d^j(0)]\gamma^\mu s^k(0)|\Lambda(P)\rangle &= \lambda_1 M\Lambda(P),\end{aligned}$$

which are estimated as

$$f_\Lambda = 6.1 \times 10^{-3}\mathrm{GeV}^2, \;\; \lambda_1 = -1.2 \times 10^{-2}\mathrm{GeV}^2$$

in the QCD sum rules approach.

Transition form factors for $\Lambda_b \to \Lambda + \gamma$ in various models

- The sum rules of the form factors are quire stable in the region $0 < q^2 < m_b^2 - 2m_b\Lambda_{QCD}$ with the variation of M_B^2. Both the form factors f_i and g_i can be fitted by the dipole form as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_i q^2/m_{\Lambda_b}^2 + b_i q^4/m_{\Lambda_b}^4},$$

with ξ_i being f_i and g_i, which can be extrapolated to the whole kinematical region $0 < q^2 < (m_{\Lambda_b} - m_\Lambda)^2$.

- Numerical results for the parameters for $\xi_i(0)$, a_i and b_i.

	twist-3			up to twist-6		
	$\xi_i(0)$	a_1	a_2	$\xi_i(0)$	a_1	a_2
f_2	$0.14^{+0.02}_{-0.01}$	$2.91^{+0.10}_{-0.07}$	$2.26^{+0.13}_{-0.08}$	$0.15^{+0.02}_{-0.02}$	$2.94^{+0.11}_{-0.06}$	$2.31^{+0.14}_{-0.10}$
$g_2(\text{GeV}^{-1})$	$-4.7^{+0.6}_{-0.6} \times 10^{-3}$	$3.40^{+0.06}_{-0.05}$	$2.98^{+0.09}_{-0.08}$	$1.3^{+0.2}_{-0.4} \times 10^{-2}$	$2.91^{+0.12}_{-0.09}$	$2.24^{+0.17}_{-0.13}$

- As a comparison, we also would like to present the results of the form factors with distribution amplitudes from COZ (Ref: V. L. Chernyak, A. A. Ogloblin and I. R. Zhitnitsky, Z. Phys. **C 42**, 569 (1989).) and FZOZ (Ref: G. R. Farrar, H. Zhang, A. A. Ogloblin and I. R. Zhitnitsky, Nucl. Phys. **B 311** (1989) 585.) up to the leading twist.

  In the COZ model, the leading twist DAs A_1 can be written as

$$\begin{aligned} A_1^{COZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\quad \times[0.26(x_3^2+x_2^2)+0.34x_1^2-0.56x_2x_3-0.24x_1(x_2+x_3)], \\ \phi_{as}(x_1,x_2,x_3) &= 120x_1x_2x_3. \end{aligned}$$

  While in the FZOZ model, the manifest expression of A_1 can be written as

$$\begin{aligned} A_1^{FZOZ}(x_1,x_2,x_3) &= -42\phi_{as}(x_1,x_2,x_3) \\ &\quad \times[0.093(x_3^2+x_2^2)+0.376x_1^2-0.194x_2x_3-0.207x_1(x_2+x_3)], \\ \phi_{as}(x_1,x_2,x_3) &= 120x_1x_2x_3. \end{aligned}$$

- The numerical results for the parameters of $\xi_i(0)$, a_i and b_i for the LCDAs of COZ model and FZOZ model have been grouped in the below Table.

	COZ			FZOZ		
	$\xi_i(0)$	a_1	a_2	$\xi_i(0)$	a_1	a_2
f_2	$0.74^{+0.06}_{-0.06}$	$2.01^{+0.17}_{-0.10}$	$1.32^{+0.14}_{-0.08}$	$0.87^{+0.07}_{-0.07}$	$2.08^{+0.15}_{-0.09}$	$1.41^{+0.11}_{-0.08}$
$g_2(\mathrm{GeV}^{-1})$	$-2.4^{+0.3}_{-0.2} \times 10^{-2}$	$2.76^{+0.16}_{-0.13}$	$2.05^{+0.23}_{-0.13}$	$-2.8^{+0.4}_{-0.2} \times 10^{-2}$	$2.80^{+0.16}_{-0.11}$	$2.12^{+0.21}_{-0.13}$

- The numerical results for the parameters of $\xi_i(0)$, a_i and b_i in the QCD sum rules can be summarized as below.

	$\xi_i(0)$	a_1	a_2
ξ_1	0.462	0.57	-0.18
ξ_2	-0.077	2.16	1.46