Some B meson decays in the pQCD approach

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纲要

- I. 介绍
- II. 理论框架和解析计算
- III. 数值结果及分析
- IV. 总结展望

I. 介绍

♣ 标准模型(SM)

- 弱、电、强三种相互作用
- 弱电统一模型+量子色动力学

♣ B物理

- 检验SM，验证QCD，研究CPV以及探寻NP
- B介子工厂
- B介子弱衰变受强相互作用影响比较大

♣ 量子色动力学(QCD)

强相互作用理论

夸克和胶子间相互作用(强相互作用)

夸克和胶子间的耦合常数$\alpha_s(Q^2)$ 对数型减小

两重性:

- 微扰
- 非微扰

因此人们不得不借助于唯象模型

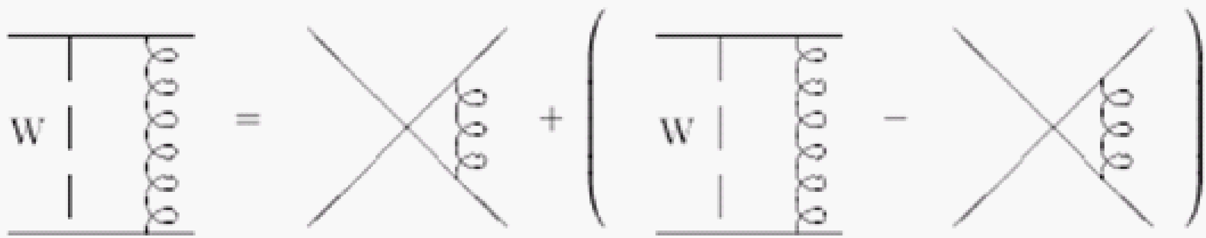
♣ B介子弱衰变

研究B介子弱衰变，一个比较大的困难是强子矩阵元的计算，目前主要有几种方法:

- Bauer等人提出的朴素因子化方案(NFA)
- Ali等人提出的推广因子化方案(GFA)
- 由Beneke等人提出的BBNS因子化方法
- 由李湘楠等人建立的pQCD方法
- Stewart等人提出的SCET方法

II. 理论框架和解析计算

♣ 低能有效哈密顿



Full theory
μ indep.

Effective theory
Low-energy < m_W

IR finite difference
High-energy» m_W

⇓ Sum $\ln(m_W/\mu)$ to all orders

Weak Hamiltonian   4-fermion operator   Wilson coefficient

H_{eff} = $O(\mu)$ - $C(\mu)$

任何衰变过程的有效哈米顿都可以表示为定域四夸克算符的线性迭加

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(\mu) Q_i^q(\mu) + h.c. \tag{1}$$

C_i 称为Wilson系数

Q_i是定域有效算符:

- 流–流算符(current–current)

$$Q_1^u = (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, \qquad Q_1^c = (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta c_\beta)_{V-A}, \tag{2}$$

$$Q_2^u = (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A}, \qquad Q_2^c = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A}, \tag{3}$$

其中α、β是$SU(3)_3$颜色指标。

- QCD企鹅算符(QCD penguins)

$$Q_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \qquad Q_4 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}, \quad (4)$$

$$Q_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \qquad Q_6 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}, \quad (5)$$

- 弱电企鹅算符(electroweak penguins)

$$Q_7 = \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \qquad Q_8 = \frac{3}{2} (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A} (6)$$

$$Q_9 = \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \qquad Q_{10} = \frac{3}{2} (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A} (7)$$

- 磁企鹅算符(magnetic penguins)

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1+\gamma_5) b_\alpha F_{\mu\nu}, \qquad Q_{8g} = \frac{g}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu} (1+\gamma_5) t^a_{\alpha\beta} b_\beta G^a_{\mu\nu}$$

♣ 强子矩阵元计算

在标准模型理论的框架下，B介子的两体衰变振幅可以写成:

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i C_i V_i \langle M_1 M_2 | O_i | B \rangle \tag{9}$$

- NFA:

$$< P_1 P_2|\mathcal{H}_{eff}|B > \;=\; < P_1|j^\mu|0 >< P_2|j_\mu|B >, \tag{10}$$

j^μ是流算符。我们定义:

$$\langle P(p)|\bar{q}\gamma^\mu L q'|0\rangle = i f_P\, p^\mu, \tag{11}$$

$$\langle P_1(p_1)|\bar{q}\gamma_\mu L b|B(p_B)\rangle = \left[(p_B+p_1)_\mu - \frac{m_B^2-m_1^2}{q^2}q_\mu\right] F_1(q^2)+\frac{m_B^2-m_1^2}{q^2}q_\mu F_0(q^2) \tag{12}$$

其中，$q=p_B-p_1$，p_1是赝标介子P_1携带的动量。

- GFA:

$$\langle Q_i(\mu)\rangle = \left[1 + \frac{\alpha_s}{4\pi}\hat{m}_s(\mu) + \frac{\alpha_{em}}{4\pi}\hat{m}_{em}(\mu)\right]\langle Q_i\rangle_{tree} \qquad (13)$$

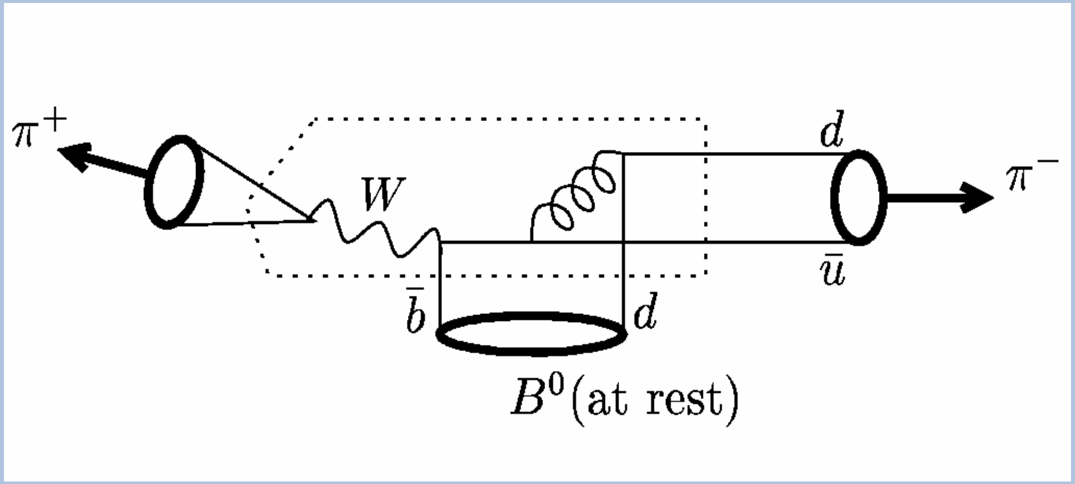
其中$\hat{m}_s(\mu)$以及$\hat{m}_{em}(\mu)$和重整化方案以及标度有关。为了补偿非因子化带来的贡献，引入了一个或者多个N_c^{eff}这个变量。N_c^{eff}不普适。

- QCDF:

从微扰QCD的基本原理出发。
非微扰：LCWF，form factor
领头阶：NFA的贡献
湮灭图、非因子化的发散处理不好

- pQCD方法

形状因子的QCD过程微扰可算。
Sudakov因子修正了端点行为，避免红外发散。
湮灭图的贡献可算。



六夸克算符(六夸克有效理论)

$$\text{Amp} \sim \int d^4k_1 d^4k_2 d^4k_3 \, \mathrm{Tr}[C(t)\Phi_B(k_1)\Phi_\pi(k_2)\Phi_{\eta^{(\prime)}}(k_3)H(k_1,k_2,k_3,t)]exp[-S(t)] \tag{14}$$

- $H(k_1,k_2,k_3,t)$硬核部分
- $C(t)$四夸克算符的有效理论

- 横动量k_T的引入

$$\frac{1}{x_2^2 x_3 M_B^4} \to \frac{1}{(x_2 M_B^2 + \mathbf{k}_{3T}^2)(x_2 x_3 M_B^2 + (\mathbf{k}_{1T} + \mathbf{k}_{3T})^2)} \tag{15}$$

- Sudakov形状因子 $exp[-S(t)]$

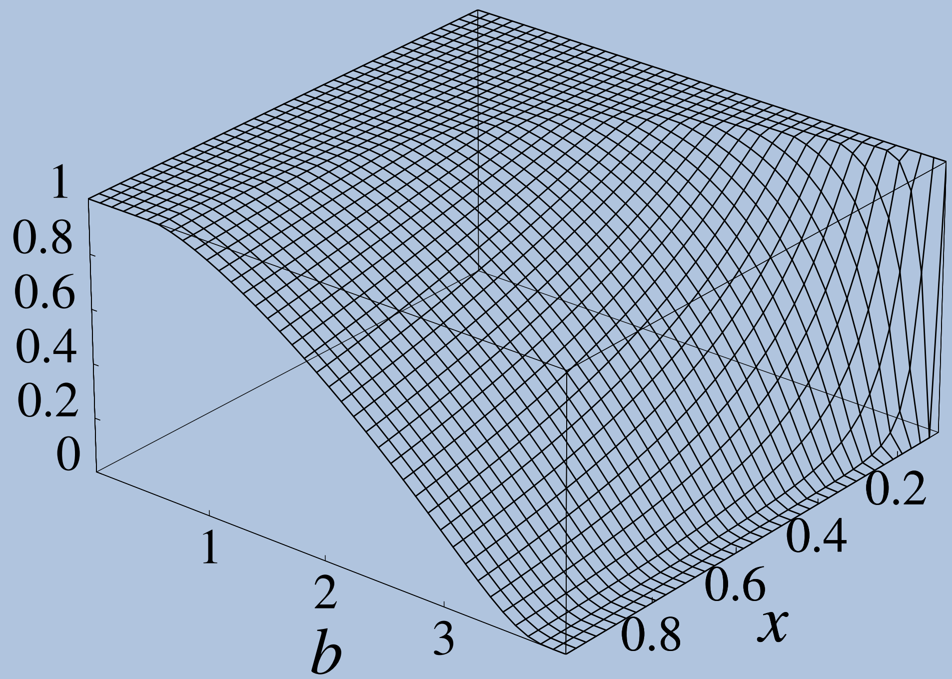


图 1: Sudakov 因子效果图

- 阈值求和$S_t(x)$

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \quad c = 0.3 \tag{16}$$

- 介子光锥波函数

$$\Phi_B(x,b) = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5\phi_B(x,b), \tag{17}$$

$$\Phi_\pi(x,b) = \frac{i\gamma_5}{\sqrt{2N_c}}\left[\not{P}\phi_\pi^A(x,b) + m_{0\pi}\phi_\pi^P(x,b) + \xi m_{0\pi}(\not{v}\ \not{n} - 1)\phi_\pi^T(x,b)\right] \tag{18}$$

$$\Phi_\eta(x,b) = \frac{i\gamma_5}{\sqrt{2N_c}}\left[\not{P}\phi_\eta^A(x,b) + m_0^\eta\phi_\eta^P(x,b) + \xi m_0^\eta(\not{v}\ \not{n} - 1)\phi_\eta^T(x,b)\right] \tag{19}$$

♣ $H(t)$微扰QCD计算

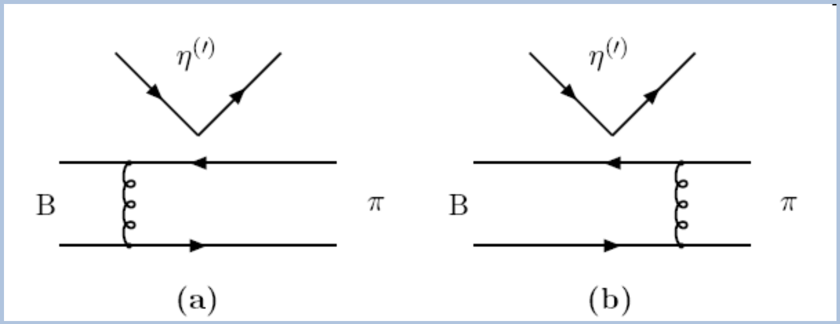


图 2: 图(a) 和(b)是$B \to \pi$跃迁的可因子化部分

$$
\begin{aligned}
F_{e\pi} \;=\;& 8\pi C_F m_B^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3\, \phi_B(x_1,b_1) \\
&\times \Big\{ \Big[(1+x_3)\phi_\pi(x_3,b_3) + (1-2x_3) r_\pi \big(\phi_\pi^p(x_3,b_3) + \phi_\pi^t(x_3,b_3)\big) \Big] \\
&\quad \times \alpha_s(t_e^1)\, h_e(x_1,x_3,b_1,b_3)\, \exp[-S_{ab}(t_e^1)] \\
&+ 2 r_\pi \phi_\pi^p(x_3,b_3) \alpha_s(t_e^2) h_e(x_3,x_1,b_3,b_1)\, \exp[-S_{ab}(t_e^2)] \Big\}\,,
\end{aligned}
$$

这里$r_\pi = m_0^\pi / m_B$; 函数h_e、能标$t_e^{1(2)}$等参见 NPB 738(2006)243-268。

可抽出$B\to\pi$的形状因子:

$$M=-i(m_B^2-m_\pi^2)f_\eta F_0(B\to\pi) \tag{20}$$

$$F_{e\pi}^{P1}=-F_{e\pi} \tag{21}$$

$$\begin{aligned}F_{e\pi}^{P2}=&-16\pi C_F m_B^4 r_\pi\int_0^1 dx_1dx_3\int_0^\infty b_1db_1b_3db_3\,\phi_B(x_1,b_1)\\ &\times\Big\{\big[\phi_\pi(x_3,b_3)+r_\pi((2+x_3)\phi_\pi^P(x_3,b_3)-x_3\phi_\pi^t(x_3,b_3)\big]\\ &\times\alpha_s(t_e^1)h_e(x_1,x_3,b_1,b_3)\exp[-S_{ab}(t_e^1)]\\ &+\big[x_1\phi_\pi(x_3,b_3)-2(x_1-1)r_\pi\phi_\pi^P(x_3,b_3)\big]\\ &\times\alpha_s(t_e^2)h_e(x_3,x_1,b_3,b_1)\exp[-S_{ab}(t_e^2)]\Big\}\end{aligned}$$

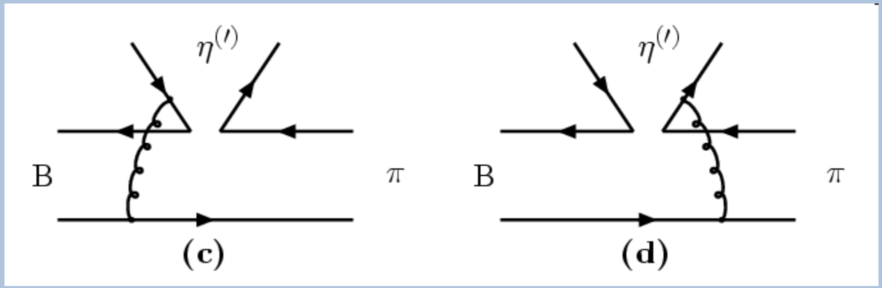


图 3: 图(c) 和(d)是旁观者夸克的非因子化部分

$$
\begin{aligned}
M_{e\pi} \;=\; & \frac{16\sqrt{6}}{3}\pi C_F m_B^4 \int_0^1 dx_1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1)\phi_\eta(x_2,b_2) \\
& \times x_3 \left[2r_\pi \phi_\pi^t(x_3,b_2) - \phi_\pi(x_3,b_2)\right] \\
& \times \alpha_s(t_f) h_f(x_1,x_2,x_3,b_1,b_2)\exp[-S_{cd}(t_f)]
\end{aligned}
$$

$$
\begin{aligned}
M_{e\pi}^{P1} &= 0, \\
M_{e\pi}^{P2} &= -M_{e\pi}
\end{aligned}
$$

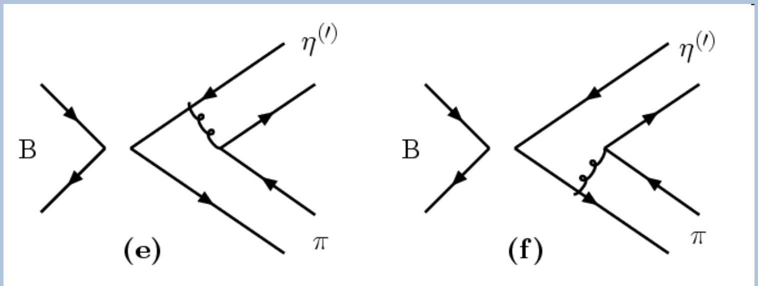


图 4: 图(e) 和(f)是湮灭的可因子化部分

$$
\begin{aligned}
F_{a\pi}^{P1} = F_{a\pi} \;\; = \;\; & 8\pi C_F m_B^4 \int_0^1 dx_2\, dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \; \{[x_3 \phi_\pi(x_3, b_3)\phi_\eta(x_2, b_2) \\
& +2 r_\eta r_\pi ((x_3+1)\phi_\pi^P(x_3, b_3) + (x_3-1)\phi_\pi^t(x_3, b_3))\phi_\eta^P(x_2, b_2)] \\
& \quad \times \alpha_s(t_e^3) h_a(x_2, x_3, b_2, b_3) \exp[-S_{gh}(t_e^3)] \\
& - [x_2 \phi_\pi(x_3, b_3)\phi_\eta(x_2, b_2) \\
& \quad +2 r_\eta r_\pi \phi_\pi^P(x_3, b_3)((x_2+1)\phi_\eta^P(x_2, b_2) + (x_2-1)\phi_\eta^t(x_2, b_2))] \\
& \quad \times \alpha_s(t_e^4) h_a(x_3, x_2, b_3, b_2) \exp[-S_{gh}(t_e^4)]\}
\end{aligned}
$$

$$
\begin{aligned}
F^{P2}_{a\pi} &= 16\pi C_F m_B^4 \int_0^1 dx_2\, dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
&\cdot \Big\{ \big[x_3 r_\pi (\phi^P_\pi(x_3,b_3) - \phi^t_\pi(x_3,b_3))\phi_\eta(x_2,b_2) + 2 r_\eta \phi_\pi(x_3,b_3)\phi^P_\eta(x_2,b_2) \big] \\
&\times \alpha_s(t_e^3) h_a(x_2,x_3,b_2,b_3) \exp[-S_{gh}(t_e^3)] \\
&+ \big[2 r_\pi \phi^P_\pi(x_3,b_3)\phi_\eta(x_2,b_2) + x_2 r_\eta (\phi^P_\eta(x_2,b_2) - \phi^t_\eta(x_2,b_2))\phi_\pi(x_3,b_3) \big] \\
&\times \alpha_s(t_e^4) h_a(x_3,x_2,b_3,b_2) \exp[-S_{gh}(t_e^4)] \Big\} \\
M_{a\pi} &= \frac{16\sqrt{6}}{3} \pi C_F m_B^4 \int_0^1 dx_1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1) \\
&\times \Big\{ - \big\{ x_2 \phi_\pi(x_3,b_2)\phi_\eta(x_2,b_2) \\
&+ r_\pi r_\eta \left[(x_2+x_3+2)\phi^P_\eta(x_2,b_2) + (x_2-x_3)\phi^t_\eta(x_2,b_2) \right] \phi^P_\pi(x_3,b_2) \\
&+ \left[(x_2-x_3)\phi^P_\pi(x_3,b_2) + (x_2+x_3-2)\phi^t_\eta(x_2,b_2) \right] \phi^t_\pi(x_3,b_2) \big\} \\
&\times \alpha_s(t_f^3) h_f^3(x_1,x_2,x_3,b_1,b_2) \exp[-S_{ef}(t_f^3)] + \big\{ x_3 \phi_\pi(x_3,b_2)\phi_\eta(x_2,b_2) \\
&- r_\pi r_\eta \left[\phi^P_\pi(x_3,b_2) \left[-(x_2+x_3)\phi^P_\eta(x_2,b_2) + (x_2-x_3)\phi^t_\eta(x_2,b_2) \right] \right. \\
&\left. - \phi^t_\pi(x_3,b_2) \left[(x_3-x_2)\phi^P_\eta(x_2,b_2) + (x_2+x_3)\phi^t_\eta(x_2,b_2) \right] \right] \big\} \\
&\times \alpha_s(t_f^4) h_f^4(x_1,x_2,x_3,b_1,b_2) \exp[-S_{ef}(t_f^4)] \Big\}
\end{aligned}
$$

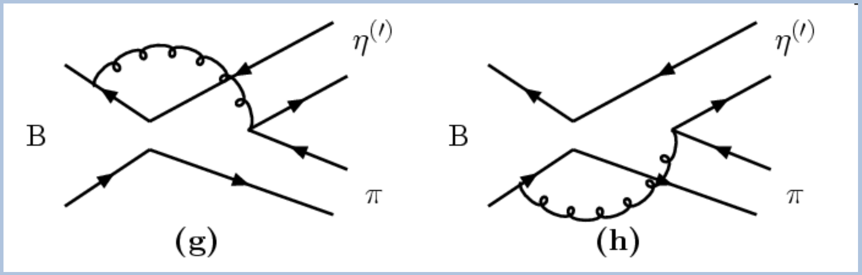


图 5: 图(g) 和(h)是湮灭的非因子化部分

$$
\begin{aligned}
M_{a\pi}^{P1} \quad = \quad & \frac{16\sqrt{6}}{3}\pi C_F m_B^4 \int_0^1 dx_1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1, b_1) \\
& \times \Big\{ \Big[(x_3-2) r_\pi \phi_\eta(x_2, b_2)(\phi_\pi^P(x_3, b_2) + \phi_\pi^t(x_3, b_2)) \\
& -(x_2-2) r_\eta \phi_\pi(x_3, b_2)(\phi_\eta^P(x_2, b_2) + \phi_\eta^t(x_2, b_2)) \Big] \\
& \cdot \alpha_s(t_f^3) h_f^3(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^3)] \\
& - \Big[x_3 r_\pi \phi_\eta(x_2, b_2)(\phi_\pi^P(x_3, b_2) + \phi_\pi^t(x_3, b_2)) \\
& -x_2 r_\eta \phi_\pi(x_3, b_2)(\phi_\eta^P(x_2, b_2) + \phi_\eta^t(x_2, b_2)) \Big] \\
& \cdot \alpha_s(t_f^4) h_f^4(x_1, x_2, x_3, b_1, b_2) \exp[-S_{ef}(t_f^4)] \Big\}
\end{aligned}
$$

$$
\begin{aligned}
M_{a\pi}^{P2} \quad = \quad & \frac{16\sqrt{6}}{3}\pi C_F m_B^4 \int_0^1 dx_1 dx_2\, dx_3 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1) \\
& \times \Big\{ \big[x_3\phi_\pi(x_3,b_2)\phi_\eta(x_2,b_2) + r_\pi r_\eta (\phi_\pi^P(x_3,b_2)((2+x_2+x_3)\phi_\eta^P(x_2,b_2) \\
& -(x_2-x_3)\phi_\eta^t) + \phi_\pi^t(-(x_2-x_3)\phi_\eta^P + (-2+x_2+x_3)\phi_\eta^t)) \big] \\
& \cdot\alpha_s(t_f^3) h_f^3(x_1,x_2,x_3,b_1,b_2) \exp[-S_{ef}(t_f^3)] \\
& + \big[-x_2\phi_\pi(x_3,b_2)\phi_\eta(x_2,b_2) - r_\pi r_\eta ((x_2+x_3)\phi_\eta^P(x_2,b_2) \\
& -(x_2-x_3)\phi_\eta^t(x_2,b_2)\phi_\pi^P(x_3,b_2) \\
& +((x_3-x_2)\phi_\eta^P(x_2,b_2) - (x_2+x_3)\phi_\eta^t(x_2,b_2))\phi_\pi^t(x_3,b_2)) \big] \\
& \cdot\alpha_s(t_f^4) h_f^4(x_1,x_2,x_3,b_1,b_2) \exp[-S_{ef}(t_f^4)] \Big\}
\end{aligned}
$$

交换π和$\eta^{(\prime)}$的位置，新的振幅表达式:

$$\phi_\pi \to \phi_\eta, \quad \phi_\pi^P \to \phi_\eta^P, \quad \phi_\pi^t \to \phi_\eta^t, \quad r_\pi \to r_\eta. \tag{22}$$

$b \to \eta^{(\prime)}$的形状因子也可以计算

- η和η'的混合:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_p & -\sin\theta_p \\ \sin\theta_p & \cos\theta_p \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \tag{23}$$

with

$$\eta_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right),$$

$$\eta_1 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$$-17^\circ \le \theta_p \le -10^\circ$$

总的振幅表达式应该为

$$
\begin{aligned}
\mathcal{M}(\pi^+\eta) \quad = \quad & \frac{1}{\sqrt{3}} F_{e\pi} \left\{ \left[\xi_u \left(C_1 + \frac{1}{3} C_2 \right) \right.\right. \\
& - \xi_t \left(\frac{7}{3} C_3 + \frac{5}{3} C_4 - 2C_5 - \frac{2}{3} C_6 - \frac{1}{2} C_7 - \frac{1}{6} C_8 + \frac{1}{3} C_9 - \frac{1}{3} C_{10} \right) \Big] f_\eta^d F_1(\theta_p) \\
& \left. - \xi_t \left(C_3 + \frac{1}{3} C_4 - C_5 - \frac{1}{3} C_6 + \frac{1}{2} C_7 + \frac{1}{6} C_8 - \frac{1}{2} C_9 - \frac{1}{6} C_{10} \right) f_\eta^s F_2(\theta_p) \right\} \\
& - F_{e\pi}^{P_2} \xi_t \left(\frac{1}{3} C_5 + C_6 - \frac{1}{6} C_7 - \frac{1}{2} C_8 \right) f_\eta^d F_1(\theta_p) \\
& + M_{e\pi} \left\{ \left[\xi_u C_2 - \xi_t \cdot \left(C_3 + 2C_4 - \frac{1}{2} C_9 + \frac{1}{2} C_{10} \right) \right] F_1(\theta_p) \right. \\
& \left. - \xi_t \left(C_4 - \frac{1}{2} C_{10} \right) F_2(\theta_p) \right\} \\
& - M_{e\pi}^{P_2} \, \xi_t \left[\left(2C_6 + \frac{1}{2} C_8 \right) F_1(\theta_p) + \left(C_6 - \frac{1}{2} C_8 \right) F_2(\theta_p) \right] \\
& + (M_{a\pi} + M_e + M_a) \left[\xi_u C_1 - \xi_t (C_3 + C_9) \right] F_1(\theta_p) \\
& - \left(M_{a\pi}^{P_1} + M_a^{P_1} \right) \, \xi_t \, (C_5 + C_7) \cdot F_1(\theta_p) \\
& + \left(F_{a\pi}^{P_2} + F_e^{P_2} + F_a^{P_2} \right) (-\xi_t) \left(C_6 + C_8 + \frac{1}{3} C_5 + \frac{1}{3} C_7 \right) F_1(\theta_p) \\
& + F_e \cdot \left\{ \left[\xi_u \left(\frac{1}{3} C_1 + C_2 \right) - \xi_t \left(\frac{1}{3} C_3 + C_4 + \frac{1}{3} C_9 + C_{10} \right) \right] F_1(\theta_p) \right\}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{6}\mathcal{M}(\pi^0\eta) \quad = \quad & F_e\left[\xi_u\left(C_1+\frac{1}{3}C_2\right)-\xi_t\left(-\frac{1}{3}C_3-C_4-\frac{3}{2}C_7-\frac{1}{2}C_8+\frac{5}{3}C_9+C_{10}\right)\right]F_1(\theta_p)\\
& -F_{e\pi}\left[\xi_u\left(C_1+\frac{1}{3}C_2\right)\cdot f_\eta^d F_1(\theta_p)\right.\\
& -\xi_t\left(\frac{7}{3}C_3+\frac{5}{3}C_4-2C_5-\frac{2}{3}C_6-\frac{1}{2}C_7-\frac{1}{6}C_8+\frac{1}{3}C_9-\frac{1}{3}C_{10}\right)\cdot f_\eta^d F_1(\theta_p)\\
& \left.-\xi_t\left(C_3+\frac{1}{3}C_4-C_5-\frac{1}{3}C_6+\frac{1}{2}C_7+\frac{1}{6}C_8-\frac{1}{2}C_9-\frac{1}{6}C_{10}\right)\cdot f_\eta^s F_2(\theta_p)\right]\\
& -M_{e\pi}\left\{\left[\xi_u C_2-\xi_t\left(C_3+2C_4-\frac{1}{2}C_9+\frac{1}{2}C_{10}\right)\right]\cdot F_1(\theta_p)\right.\\
& \left.-\xi_t\left(C_4-\frac{1}{2}C_{10}\right)F_2(\theta_p)\right\}\\
& -M_{e\pi}^{P_2}\left[-\xi_t\left(2C_6+\frac{1}{2}C_8\right)F_1(\theta_p)-\xi_t\left(C_6-\frac{1}{2}C_8\right)F_2(\theta_p)\right]\\
& -(F_{a\pi}^{P_2}+F_a^{P_2}+F_e^{P_2})\left[-\xi_t\left(C_6+\frac{1}{3}C_5-\frac{1}{2}C_8-\frac{1}{6}C_7\right)\right]\cdot F_1(\theta_p)\\
& +(M_{a\pi}+M_a+M_e)\left[\xi_u C_2-\xi_t\left(-C_3+\frac{1}{2}C_9+\frac{3}{2}C_{10}\right)\right]F_1(\theta_p)\\
& +\left(M_a^{P_1}+M_{a\pi}^{P_1}\right)\ \xi_t\ \left(C_5-\frac{1}{2}C_7\right)F_1(\theta_p)\\
& -\frac{3}{2}\left(M_{a\pi}^{P_2}+M_a^{P_2}+M_e^{P_2}\right)\xi_t C_8 F_1(\theta_p)+F_{e\pi}^{P_2}\xi_t\left(\frac{1}{3}C_5+C_6-\frac{1}{6}C_7-\frac{1}{2}C_8\right)F_1(\theta_p)
\end{aligned}
$$

其中，$\xi_u = V_{ub}^* V_{ud}$, $\xi_t = V_{tb}^* V_{td}$, 还有

$$F_1(\theta_p) = -\sin\theta_p + \cos\theta_p/\sqrt{2},$$
$$F_2(\theta_p) = -\sin\theta_p - \sqrt{2}\cos\theta_p, \tag{24}$$

是混合因子

$$f_\eta^d,\ f_\eta^s \longrightarrow f_{\eta'}^d,\ f_{\eta'}^s,$$
$$F_1(\theta_p) \longrightarrow F_1'(\theta_p) = \cos\theta_p + \frac{\sin\theta_p}{\sqrt{2}},$$
$$F_2(\theta_p) \longrightarrow F_2'(\theta_p) = \cos\theta_p - \sqrt{2}\sin\theta_p. \tag{25}$$

未考虑η'可能存在的胶子成分。

III. 数值结果及分析

$$\mathcal{M} = V_{ub}^* V_{ud} T - V_{tb}^* V_{td} P = V_{ub}^* V_{ud} T \left[1 + z e^{i(\alpha+\delta)}\right], \quad (26)$$

其中，

- $z = \left|\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right| \left|\frac{P}{T}\right|$是企鹅图和树图贡献的比率
- $\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$ 代表弱相位(幺正三角形的一个角)
- δ是树图T和企鹅图P的相对相位。

z和δ 可以用pQCD方法计算得到:

$$z(\pi^+\eta) = 0.33, \qquad \delta(\pi^+\eta) = -136^\circ,$$
$$z(\pi^+\eta') = 0.25, \qquad \delta(\pi^+\eta') = -130^\circ.$$

$$\overline{\mathcal{M}} = V_{ub}V_{ud}^{*}T - V_{tb}V_{td}^{*}P = V_{ub}V_{ud}^{*}T\left[1 + ze^{i(-\alpha+\delta)}\right].$$

$B^0 \to \pi\eta^{(\prime)}$的CP平均分支比是，

$$Br = (|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2)/2 = |V_{ub}V_{ud}^{*}T|^2\left[1 + 2z\cos\alpha\cos\delta + z^2\right],$$

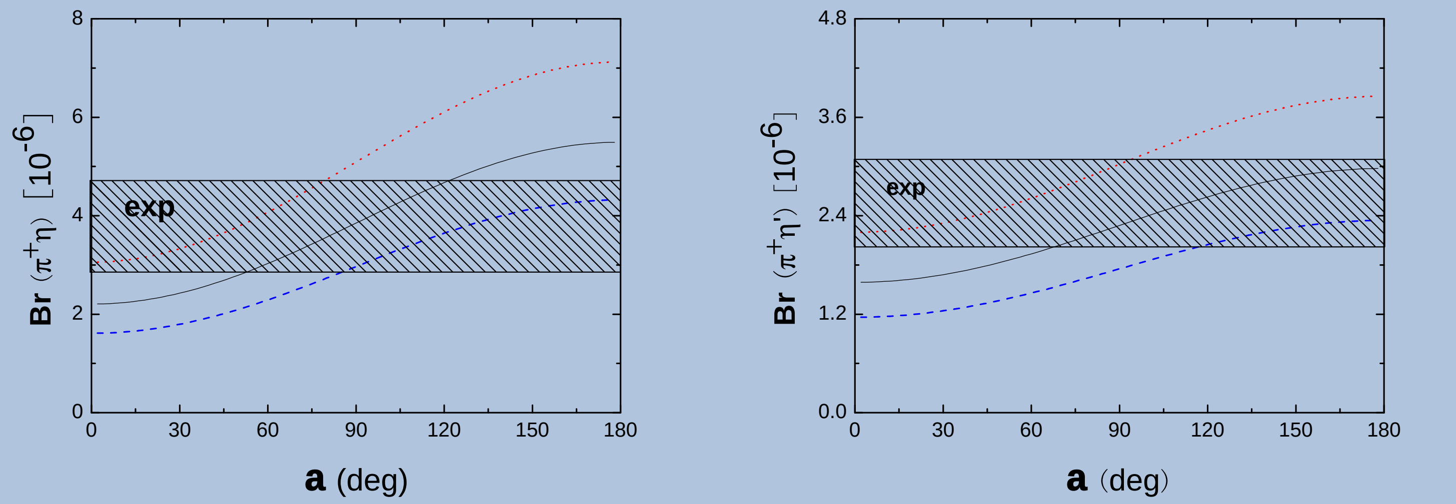


图 6: 当$m_0^\pi = 1.4$ GeV，$\theta_p = -17°$时，$B^+ \to \pi^+\eta^{(\prime)}$衰变分支比(单位：$10^{-6}$)，分别在$\omega_b = 0.36$ GeV (点线), 0.40 GeV (实线)和0.44 GeV(点划线)时随α的变化曲线。阴影区域是实验范围

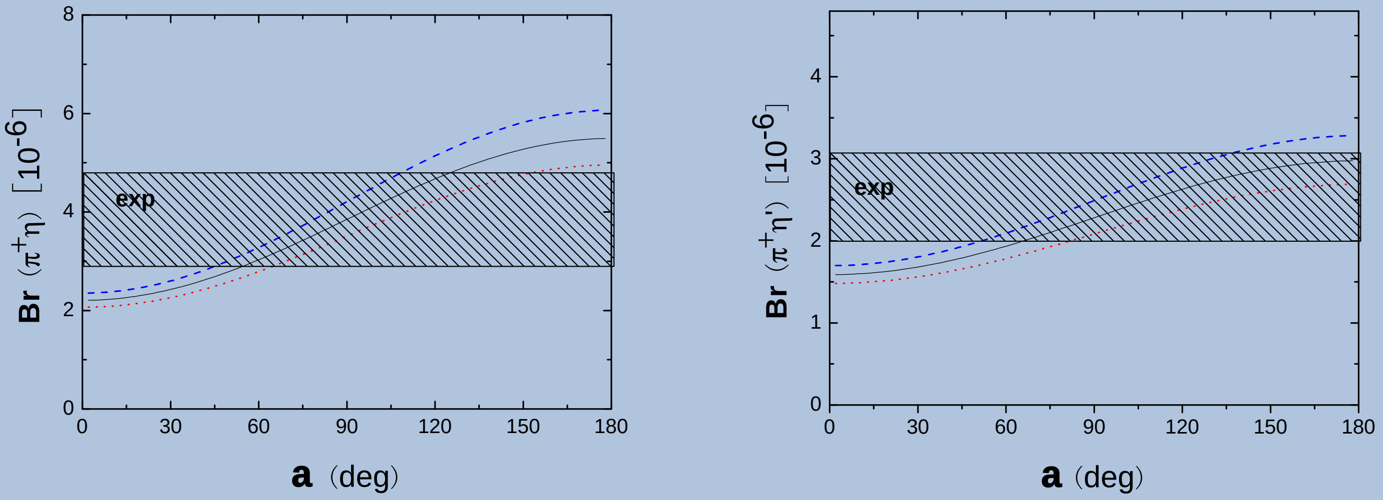


图 7: 当$\omega_b = 0.4$ GeV，$\theta_p = -17°$时，$B^+ \to \pi^+\eta^{(\prime)}$衰变分支比(单位：$10^{-6}$)，分别在$m_0^\pi = 1.3$ GeV(点线), 1.4 GeV (实线) 和1.5 GeV (点划线)时随α的变化曲线。阴影区域是实验范围

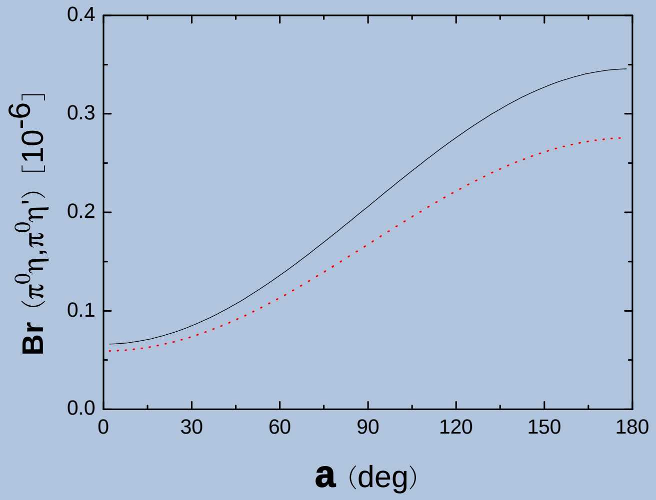


图 8: 当$m_0^\pi = 1.4$ GeV, $\theta_p = -17°$, $\omega_b = 0.40$ GeV 时，$\pi^0\eta$ (实线) 和$\pi^0\eta'$ (点线)衰变分支比(单位：10^{-6})随α的变化曲线

QCD因子化的理论预言:

$$
\begin{aligned}
Br(\ B^+ \to \pi^+\eta) &= (4.7^{+2.7}_{-2.3}) \times 10^{-6}, \\
Br(\ B^+ \to \pi^+\eta') &= (3.1^{+1.9}_{-1.7}) \times 10^{-6}, \\
Br(\ B^0 \to \pi^0\eta) &= (0.28^{+0.48}_{-0.28}) \times 10^{-6}, \\
Br(\ B^0 \to \pi^0\eta') &= (0.17^{+0.33}_{-0.17}) \times 10^{-6},
\end{aligned}
$$

pQCD下分支比，在$\theta_p = -17°$时，

$$Br(\ B^+ \to \pi^+ \eta) \ = \ \left[4.1^{+1.3}_{-0.9}(\omega_b)^{+0.4}_{-0.3}(m_0^\pi)^{+0.6}_{-0.5}(\alpha)\right] \times 10^{-6},$$
$$Br(\ B^+ \to \pi^+ \eta') \ = \ \left[2.4^{+0.8}_{-0.5}(\omega_b) \pm 0.2(m_0^\pi) \pm 0.3(\alpha)\right] \times 10^{-6},$$
$$Br(\ B^0 \to \pi^0 \eta) \ = \ \left[0.23^{+0.04}_{-0.03}(\omega_b)^{+0.04}_{-0.03}(m_0^\pi) \pm 0.05(\alpha)\right] \times 10^{-6},$$
$$Br(\ B^0 \to \pi^0 \eta') \ = \ \left[0.19 \pm 0.02(\omega_b) \pm 0.03(m_0^\pi)^{+0.04}_{-0.05}(\alpha)\right] \times 10^{-6},$$

当$\theta_p = -10°$时，

$$Br(\ B^+ \to \pi^+ \eta) \ = \ \left[3.3^{+1.0}_{-0.8}(\omega_b) \pm 0.3(m_0^\pi) \pm 0.4(\alpha)\right] \times 10^{-6},$$
$$Br(\ B^+ \to \pi^+ \eta') \ = \ \left[3.2^{+1.1}_{-0.7}(\omega_b) \pm 0.3(m_0^\pi) \pm 0.4(\alpha)\right] \times 10^{-6},$$
$$Br(\ B^0 \to \pi^0 \eta) \ = \ \left[0.17 \pm 0.02(\omega_b) \pm 0.02(m_0^\pi)^{+0.03}_{-0.04}(\alpha)\right]) \times 10^{-6},$$
$$Br(\ B^0 \to \pi^0 \eta') \ = \ \left[0.28^{+0.04}_{-0.02}(\omega_b) \pm 0.04(m_0^\pi) \pm 0.05(\alpha)\right] \times 10^{-6},$$

直接CP破坏$\mathcal{A}_{CP}$:

$$\mathcal{A}_{CP}^{dir} = \frac{|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2}{|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2} = \frac{2z\sin\alpha\sin\delta}{1+2z\cos\alpha\cos\delta+z^2}, \tag{27}$$

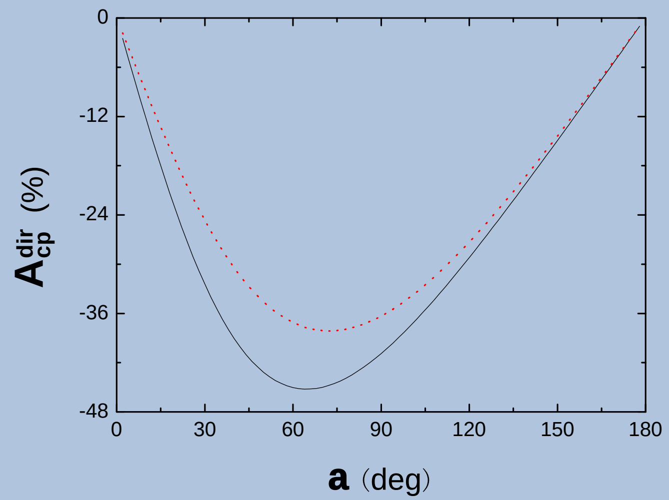


图 9: $B^+ \to \pi^+\eta$ (实线) 和$B^+ \to \pi^+\eta'$ (点线)衰变的直接CP破坏(百分比)随α的变化曲线。

pQCD结果:

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta) = \left(-37^{+8}_{-6}(\alpha) \pm 4(\omega_b)^{+0}_{-1}(m_0^\pi)\right) \times 10^{-2}, \tag{28}$$

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta') = \left(-33^{+6}_{-4}(\alpha)^{+4}_{-6}(\omega_b)^{+0}_{-2}(m_0^\pi)\right) \times 10^{-2}, \tag{29}$$

增加了各种输入参数的default values，在QCDF因子化方法下得出直接CP破坏$\mathcal{A}_{CP}^{dir}(B \to \pi^\pm \eta^{(\prime)})$的结果:

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta) = \left(-14.9^{+4.9}_{-5.4}{}^{+8.3}_{-7.4}{}^{+1.3}_{-0.8}{}^{+17.4}_{-17.3}\right) \times 10^{-2}, \tag{30}$$

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta') = \left(-8.6^{+2.8}_{-3.1}{}^{+10.5}_{-9.0}{}^{+0.7}_{-0.7}{}^{+20.4}_{-20.4}\right) \times 10^{-2}, \tag{31}$$

采用 “set S_4” 输入参数,QCDF的理论中心值就同时反号了:

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta) = 5.6 \times 10^{-2}, \tag{32}$$

$$\mathcal{A}_{CP}^{dir}(B^\pm \to \pi^\pm \eta') = 11.1 \times 10^{-2}. \tag{33}$$

B_0介子

$$\begin{aligned} A_{CP} &\equiv \frac{\Gamma\left(\overline{B_d^0}(\Delta t)\to f_{CP}\right)-\Gamma\left(B_d^0(\Delta t)\to f_{CP}\right)}{\Gamma\left(\overline{B_d^0}(\Delta t)\to f_{CP}\right)+\Gamma\left(B_d^0(\Delta t)\to f_{CP}\right)} \\ &= A_{CP}^{dir}\cos(\Delta m\Delta t)+A_{CP}^{mix}\sin(\Delta m\Delta t), \end{aligned} \tag{34}$$

$$\mathcal{A}_{CP}^{dir}=\frac{|\lambda_{CP}|^2-1}{1+|\lambda_{CP}|^2}, \qquad A_{CP}^{mix}=\frac{2Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}, \tag{35}$$

其中，CP破坏的参数λ_{CP}为:

$$\lambda_{CP}=\frac{V_{tb}^*V_{td}\langle\pi^0\eta^{(\prime)}|H_{eff}|\overline{B}^0\rangle}{V_{tb}V_{td}^*\langle\pi^0\eta^{(\prime)}|H_{eff}|B^0\rangle}=e^{2i\alpha}\frac{1+ze^{i(\delta-\alpha)}}{1+ze^{i(\delta+\alpha)}}. \tag{36}$$

z 和δ 是可算的，则可求出直接和混合CP破坏，A_{CP}^{dir}、A_{CP}^{mix}。

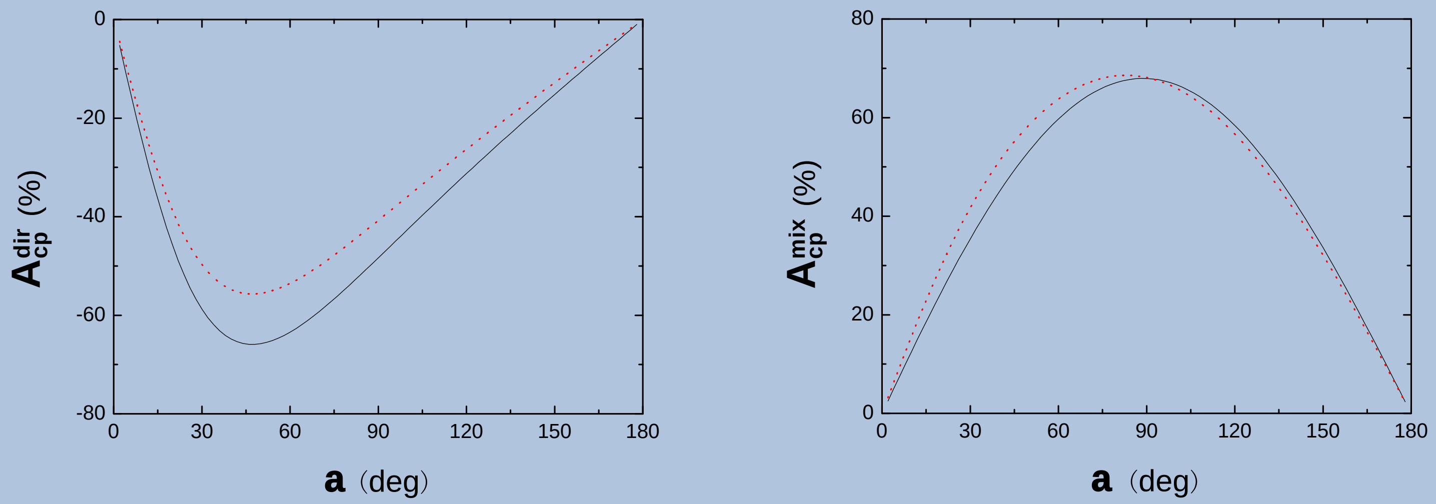


图 10: $B^0 \to \pi^0\eta$ (实线) 和$B^0 \to \pi^0\eta'$ (点线) 衰变的直接和混合CP破坏A_{CP}^{dir}(百分比)随α角的变化关系

$$\mathcal{A}_{CP}^{dir}(B^0 \to \pi^0\eta) = \left(-42^{+9}_{-12}(\alpha)^{+3}_{-2}(\omega_b)^{+1}_{-3}(m_0^\pi)\right) \times 10^{-2}, \quad (37)$$

$$\mathcal{A}_{CP}^{dir}(B^0 \to \pi^0\eta') = \left(-36^{+10}_{-9}(\alpha)^{+2}_{-1}(\omega_b)^{+2}_{-3}(m_0^\pi)\right) \times 10^{-2}, \quad (38)$$

$$\mathcal{A}_{CP}^{mix}(B^0 \to \pi^0\eta) = \left(67^{+0}_{-9}(\alpha)^{+5}_{-6}(\omega_b)^{+1}_{-2}(m_0^\pi)\right) \times 10^{-2}, \quad (39)$$

$$\mathcal{A}_{CP}^{mix}(B^0 \to \pi^0\eta') = \left(67^{+0}_{-9}(\alpha)^{+4}_{-6}(\omega_b)^{+1}_{-3}(m_0^\pi)\right) \times 10^{-2}, \quad (40)$$

主要误差来自于$\omega_b = 0.4 \pm 0.05$ GeV, $m_0^\pi = 1.4 \pm 0.1$ GeV 和$\alpha = 100^\circ \pm 20^\circ$的变化。

为了作一下比较，我们再看看QCD因子化计算$\mathcal{A}_{CP}^{dir}(B^0 \to \pi^0\eta^{(\prime)})$的大小:

$$\mathcal{A}_{CP}^{dir}(B^0 \to \pi^0\eta) = \left(-17.9^{+5.2}_{-4.1}{}^{+7.9}_{-14.1}{}^{+1.2}_{-1.4}{}^{+33.4}_{-32.9}\right) \times 10^{-2}, \quad (41)$$

$$\mathcal{A}_{CP}^{dir}(B^0 \to \pi^0\eta') = \left(-19.2^{+5.5}_{-4.3}{}^{+7.7}_{-7.8}{}^{+4.1}_{-3.3}{}^{+35.7}_{-35.8}\right) \times 10^{-2}, \quad (42)$$

我们对时间变量t求积分，就可以得到$B^0 \to \pi^0\eta^{(\prime)}$衰变的总的CP破坏:

$$A_{CP} = \frac{1}{1+x^2} A_{CP}^{dir} + \frac{x}{1+x^2} A_{CP}^{mix}, \tag{43}$$

这里，由$B^0 - \overline{B}^0$混合产生的$x = \Delta m/\Gamma = 0.771$。图11所示的分别是$B^0 \to \pi^0\eta$ (实线) 和$B^0 \to \pi^0\eta'$ (点线)的总CP破坏A_{CP} 随α的变化曲线。数值结果为:

$$\mathcal{A}_{CP}^{tot}(B^0 \to \pi^0\eta) = \left(+3.7^{+3.5}_{-7.3}(\alpha)^{+4.6}_{-3.9}(\omega_b)^{+0.3}_{-1.0}(m_0^\pi)\right) \times 10^{-2}, \tag{44}$$

$$\mathcal{A}_{CP}^{tot}(B^0 \to \pi^0\eta') = \left(+7.7^{+1.9}_{-5.1}(\alpha)^{+0.3}_{-0.8}(\omega_b)^{+1.9}_{-5.1}(m_0^\pi)\right) \times 10^{-2}. \tag{45}$$

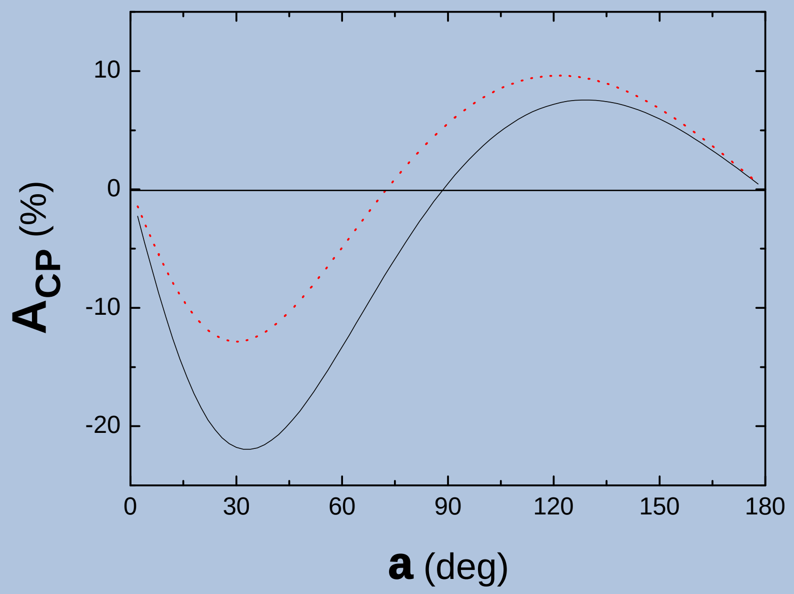


图 11: $B^0 \to \pi^0\eta$ (实线) 和$B^0 \to \pi^0\eta'$ (点线) 衰变的总CP破坏A_{CP}^{tot}(百分比)随α角的变化关系

♣ η'可能胶子成分的效应

前面我们计算$B \to \pi\eta'$衰变的时候并未考虑由η'中可能胶子的成分所引入的分支比和CP破坏大小。η'衰变振幅$\mathcal{M}'$ 中含有由胶子成分引入的新振幅。这部分新振幅对η'中的对应$q\bar{q}(q = u, d, s)$成分的振幅可能相长，可能相消。这样导致分支比也相应的增加和减少。

从计算结果可以看出用pQCD计算$Br(B^+ \to \pi^+\eta')$的结果和测量值，在1个标准偏差范围内符合得很好。而且，对$B \to \rho\eta^{(\prime)}$的分支比预言和实验数据高度一致。因此，我们认为$\eta'$中胶子混合态应该很小，并不像以前说的那么重要。

至于$B \to \pi\eta'$衰变中的CP破坏，η'中胶子成分对它的贡献相互相消。

VI. 总结和展望

pQCD方法的主要误差来源:

- 介子波函数
- 手征增强因子
- 衰变常数
- 高阶修正

pQCD方法的优点:

- 直接引入横动量，有Sudakov压低，无端点发散
- 形状因子可算
- 能计算非因子化图，湮灭图的贡献

结果总结

形状因子在pQCD下计算：$F_{0,1}^{B\to\pi}(0)=0.30$, $F_{0,1}^{B\to\eta}(0)=0.15$,$F_{0,1}^{B\to\eta'}(0)=0.14$

分支比的理论预言：B^+介子$\sim\mathcal{O}(10^{-6})$，B^0介子$\sim\mathcal{O}(10^{-7})$

直接和间接CP破坏的理论预言比较大，总的CP破坏在$3\%\sim 8\%$

通过理论预言和实验数据比较我们认为η'中胶子混合态应该很小，并不像以前说的那么重要

继续的工作

理论本身还需进一步完善，各种理论之间的分歧有待解决

双圈图的计算

谢谢大家！